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Weight Restrictions on Geography Variables in the DEA Benchmarking Model for Norwegian Electricity Distribution Companies

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Summary

We examine weight restrictions in the DEA model for distribution networks, taking as the starting point the NVE model with one input, total cost, and several outputs. In the unrestricted DEA models, we notice large differences in absolute and relative shadow prices, and for some companies, extreme weight on “geography” variables in the cost norms. There seems to be a tendency that companies with a large weight on geographic variables and / or a low weight on transported energy and customers become super efficient. This seems unreasonable, and one remedy may be to restrict prices / weights for individual outputs, or combinations of outputs. We consider absolute, relative and virtual weight restrictions, and show how to formulate the LP problems and how to interpret the restrictions. We discuss the relative price restrictions suggested for geography and high voltage variables by NVE (2008), and consider an alternative approach, using virtual weight restrictions on the combination of the three geography variables; forest, snow, and coast. Comparing the effects of the virtual approach to the relative, we notice that with relative weight restrictions, more companies are affected, but to a lesser extent. An important task when introducing weight restrictions in the DEA analyses is to determine the specific limits on the weights. Finding reasonable limits, depends on which type of weight restrictions that are considered, and should be based on knowledge of cost and technology in the industry. An advantage of the virtual weight restrictions is that they are on a more aggregated level than the relative ones, and it may be easier to establish limits on the overall effects on the total cost norm from a subset of outputs, rather than reasonable pair-wise comparisons of outputs weights. Finally, the report discusses implementation of DEA models with weight restrictions, and gives a short overview of available software.

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1. Introduction

In the Norwegian electricity sector, network companies are regulated by means of a yardstick model. Annual revenue caps are determined for individual companies based on a combination of actual cost and cost norms, according to the following formula:

$$IR = K + \rho(K^* - K) = \rho K^* + (1-\rho)K \,,$$

where IR is the revenue cap, K is the actual cost, K^* is the cost norm, and $\rho \in [0,1]$ is a factor that specifies the strength of the incentives in the yardstick model, i.e. the weight that is attributed to the cost norm. For 2007 and 2008, ρ is equal to 0.5, however, it is supposed to increase to 0.6. Actual cost and cost norms are updated annually, although, in practice, due to accounting procedures and the need for securing the quality of the data, there is a time lag in the application of cost data. At present (since 2007) the cost data used for calculating actual cost and analyzing relative efficiency is 2 years, i.e. the input for calculating cost data and performance for year t, is data from year t-2.

More specifically, actual total company cost K estimated for year t consists of a combination of registered and calculated costs, based on accounting values in year t-2. Operation and maintenance costs (OM) from year t-2 are adjusted for inflation (KPI), depreciation (DEP) equals the accounting values in year t-2, while network losses (NL) are found by taking the losses in MWh in year t-2 and multiplying by an average area price (collected from Nord Pool Spot) for year t. The cost of capital is found by multiplying the book value (BV) of the company assets at 31.12 in year t-2 by the NVE rate of return, r_{NVE}. This regulated rate of return is determined annually, based on a risk free rate of return and a risk premium. Finally, the value of lost load (VOLL) is added to the cost base. VOLL is calculated as lost load times a price, with different prices for various customer groups.

For distribution companies and regional transmission companies, the cost norm, K^*, is calculated based on relative efficiency scores found by DEA (Data Envelopment Analysis). There are separate DEA models for distribution functions on the one hand and regional transmission / central grid functions on the other hand[1]. The applied DEA models are cost efficiency models with CRS (constant returns to scale) and a single input equal to total cost K, i.e. both operating and capital expenditures are included in the performance evaluations. A variant of super efficiency is implemented such that efficiency scores may be higher than 100 % (a company that performs better than the other companies and improves over time). When evaluating relative efficiency with DEA, average (industry) efficiency will depend on implementation details like for instance the number of evaluated companies (the size of the data set), the number and specific choice of outputs, assumptions about scale efficiency, and whether super efficiency is modeled or not. In order to secure efficiency improvements over time and an attractiveness of the industry to investors and employees, it is important that particularly efficient companies can earn more then the normal rate of return. Thus, the efficiency scores are calibrated such that the representative company earns the normal rate of return. Since 2007, the representative company is the averagely efficient company, and consequently, the efficiency numbers found from the DEA analyses are calibrated such that the cost weighted average efficiency score is 100 % (Bjørndal and Bjørndal (2006b) and NVE (2006ab)). This also implies that $\sum K = \sum K^{**}$, where K^{**} is the calibrated / normalized cost norm.

Finally, due to the time lag in the use of accounting data, new investments must be compensated in order to earn the normal rate of return in a representative company. This is accomplished through an adjustment parameter, *JP* (this parameter and its use is discussed in Bjørndal et al. 2008[2]). The formula for establishing the revenue of a company in year t can then be written as:

$$IR_t = \rho K_{t-2}^{**} + (1-\rho)K_{t-2} + JP = \rho E_{t-2}^{*} K_{t-2} + (1-\rho)K_{t-2} + JP$$

where K_{t-2} is the price adjusted cost base from year t-2, E_{t-2}^{*} is the calibrated efficiency score of the company, and K_{t-2}^{**} is the corresponding calibrated cost norm.

[1] Also for Statnett, the system operator and main owner of the Central grid, revenue is regulated. Statnett is also benchmarked relative to other European system operators (ECOM / ECOM+).

[2] In Bjørndal et al. (2008) we discuss the combined effect of normalization of efficiency scores and adjustment parameter for new investments, and that the compensation for time lags is taken back in a second calibration procedure. In this report we will not discuss this issue any further.

The DEA model used for efficiency analyses has a single input equal to total cost, but many outputs, that can be interpreted as cost drivers. Some of the outputs are "product attributes", like delivered energy and the number of customer connections. However, others are exogenous or endogenous factors that are included in order to take into account differences in the "degree of difficulty" in providing network services in various license areas. Some of these outputs are in fact input factors, and in general they are "proxies" for environmental or geographic cost drivers related to customer density, topology, weather conditions, and similar. After the introduction of the new regulation model from 2007, it has been a worry that non-product outputs are allowed too large weights in the analyses, and that the consequence is overcompensation of companies that are "unusual" (having few peers to compare with) rather than efficient.

In this report, we discuss methods to alleviate this problem, with special focus on weight restrictions on "geography" factors in the DEA model for distribution networks. In section 2 the DEA model for distribution networks is described, and applied to industry data from 2005 and 2006. In section 3 we describe different versions of weight restrictions, and what interpretation they may have in a DEA model with cost as the only input factor. In section 4 we outline the restrictions proposed by NVE (2008), and we evaluate and suggest a revised version of those restrictions that are related to the geography variables. In section 5 we propose alternatives, and we compare them to the restrictions in the NVE proposal. In section 6 we discuss how to determine the specific limits on weights, and in section 7 we touch upon some implementation issues, including available software. Finally, conclusions and recommendations are found in section 8.

2. The DEA model for benchmarking distribution companies

2.1 Model specification

For distribution companies, the efficiency scores for year *t* are estimated using an input-oriented CRS model with data from year *t*-2[3]. The model has total cost, including capital costs, as the only input, and 9 output variables, as shown in figure 2.1 below.

Variable	**Unit of measurement**
Energy delivered	MWh
Customers (except cottages)	No. of customers
Cottage customers	No. of customers
High voltage lines	Kilometers
Net stations (transformers)	No. of stations
Interface	Cost weighted sum of equipment in the interface between the distribution network and the regional transmission network
Forest	Proportion (0-100) of area with high-growth forest × HV-lines through air (kilometers)
Snow	Average precipitation as snow (mm) × HV-lines through air (kilometers)
Coast	Average wind speed (m/s) / Average distance to coast (meters) × HV-lines through air (kilometers)

Figure 2.1: Output variables of the DEA model

The output variables do not, with the exception of energy delivered and the number of customers connected, measure direct outputs from the production activity of the distribution companies, but rather represent structural and environmental conditions that may influence the cost of the companies. Three of the variables (HV-lines, net stations, and interface) are in fact input variables. Their role in the DEA model, however, is to

[3] NVE uses an average over several years to represent the VOLL cost in their DEA analyses. For the 2008 revenue limit calculations, the average is taken over the years 2003-2006. However, final efficiency scores for inefficient companies, i.e. those with an efficiency score of less than 100 %, are adjusted to reflect the actual VOLL cost in year *t*-2. In practice this is done by replacing average VOLL cost with the actual VOLL cost for year *t*-2, and then recalculating the efficiency score for each company. Although this adjustment can have a significant effect on the efficiency scores of individual companies, the effect is not systematic, and we have therefore chosen to use the average VOLL cost in our calculations.

represent demographical and topological conditions, as well as transmission functions, that influence the costs of a particular company, and for which a better representation could not be found. The last three variables (forest, snow, coast) describe environmental conditions that may influence the cost of the companies, and are the only variables that are not based on data reported by the companies.

The selection of output variables was one of the most challenging issues when the new regulation model was developed prior to its introduction in 2007. In NVE (2006a), the regulator formulated three criteria that should be met if an output variable was to be included in the model: Firstly, the variable should have a solid "theoretical and practical" foundation. Secondly, it should have a statistically significant effect on company costs in SFA model test, as well as on the DEA efficiency in OLS regression tests. Thirdly, the variable should also be statistically significant in the so-called "Banker test", see Banker (1993). Hence, although a large number of candidate variables were considered initially, the final set of variables was determined mainly based on statistical tests. For example, a variable representing low voltage lines was rejected based on the Banker test, whereas the high voltage line variable passed the test and is included in the model. Since the statistical correlation between the two variables is high, this may seem quite unproblematic. However, since the companies to some extent will view the two types of lines as substitutes, the omission of one of them on the output side of the model may tilt the investment incentives of the companies in favor of the other one. The fact that the DEA model to some extent is "incomplete", i.e., that relevant output variables have been omitted because they are correlated with variables that are included, must be taken into account when considering relative weight restrictions such as in NVE (2008). We will come back to this issue in section 5 where we propose virtual weight restrictions as an alternative to the relative restrictions in NVE (2008).

Companies are allowed to be super efficient, i.e. efficiency scores may exceed 100 %. In order to avoid very high efficiency scores, super efficient companies are re-evaluated against a data set from the year(s)[4] preceding *t*-2. The DEA model in the second step includes data for the company itself, hence a company can only appear as super efficient if it has improved its performance relative to the previous year(s). In this report, we only

[4] For 2007, which was the first year of the new regulation model, the second step DEA analyses were based on data from 2004. For 2008, the second step used average data from 2004-2005.

consider the DEA analyses performed in the first step, i.e., a super efficiency model based on data from year *t*-2.

2.2 Efficiency scores

In figure 2.2 we have plotted the efficiency scores for 2005 and 2006, and we see that for the 127 companies in the data sets[5] the efficiency scores lie in the range between 60 % and 140 %, with a cost weighted industry average somewhat above 90 %. We also notice that, although the efficiency scores for individual companies in the two years seem to be highly correlated, there is considerable variation from one year to another.

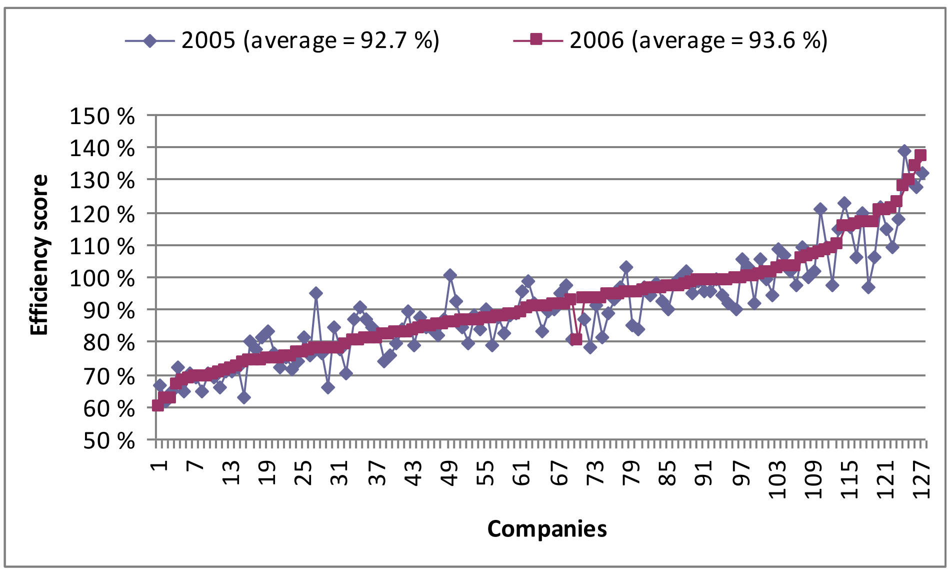

Figure 2.2: Efficiency scores for 2005 and 2006

In figures 2.3-2.7 we show that the effects on the efficiency scores from introducing geography variables as outputs are considerable. This is so for each variable, as well as the combined effect.

[5] There are 134 and 136 companies in the data sets for 2005 and 2006, respectively, but we have omitted some of them because of data quality issues. The omitted companies constitute less than 1 % of the total cost base for the industry.

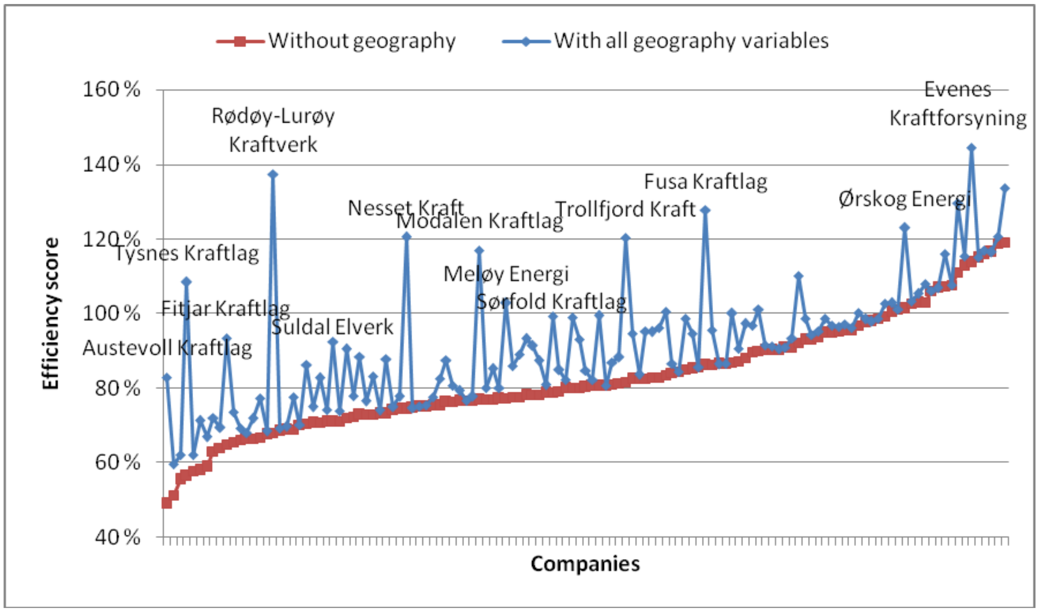

Figure 2.3: Total effect from geography outputs – forest, snow, coast (name shown if effect is at least 20 %-points)

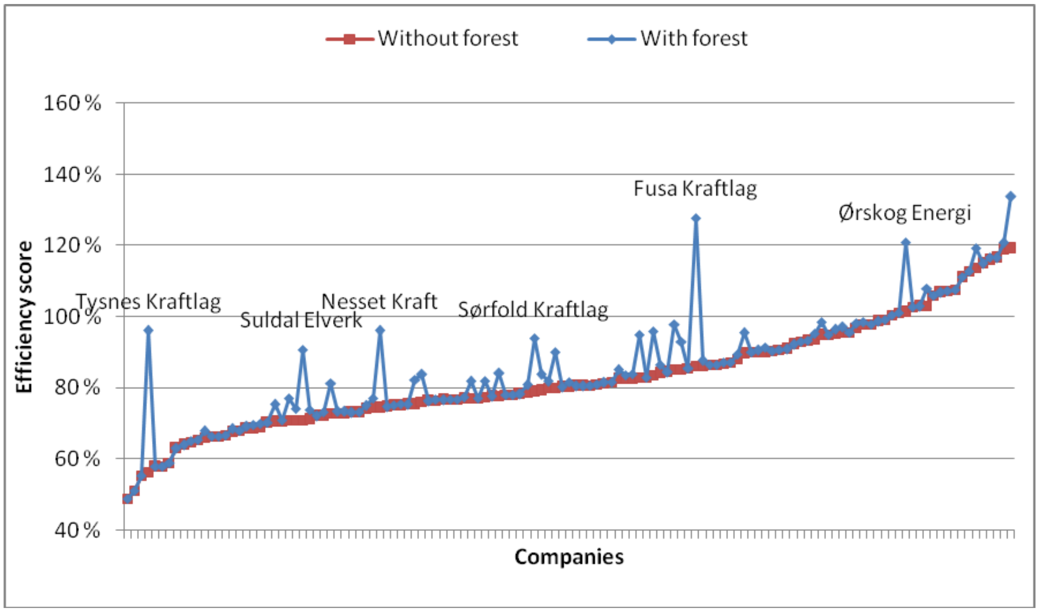

Figure 2.4: Effect of forest variable (name shown if effect is at least 15 %-points)

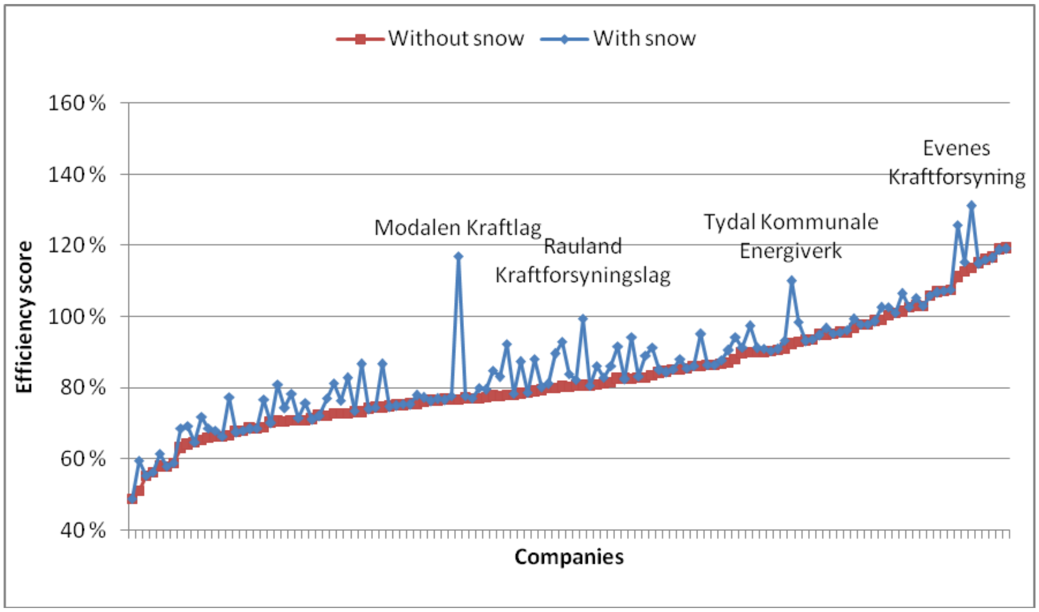

Figure 2.5: Effect of snow variable (name shown if effect is at least 15 %-points)

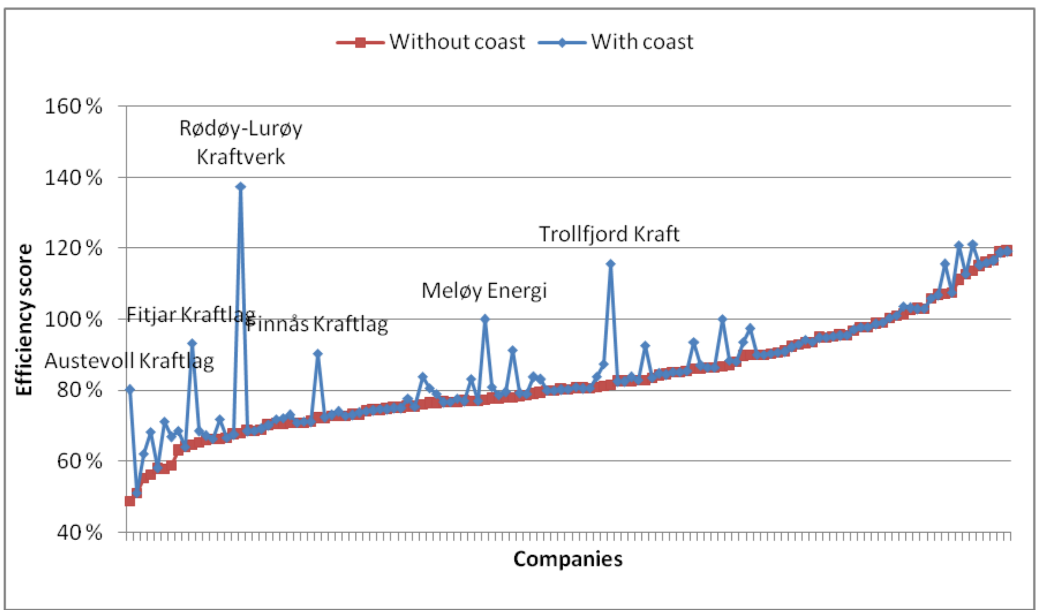

Figure 2.6: Effect of coast variable (name shown if effect is at least 15 %-points)

	Without geography	With forest	With snow	With coast	With all geography var.
Weighted average	89.8 %	91.1 %	91.5 %	91.2 %	93.6 %
Simple average	82.8 %	85.5 %	86.2 %	86.0 %	91.5 %

Figure 2.7: Average effects of geography variables

2.3 Marginal values of outputs – output weights

Looking at the shadow prices of the outputs for different companies, we notice huge differences from one company to another. This is true both for the absolute prices, for the relative prices, and for the combined effect of prices and outputs on the objective function (the total cost norm).

Absolute price levels

In figure 2.8 we present some statistics for the output prices or weights. For a particular output, its price or weight can be interpreted as the marginal change in the company's cost norm[6], given that the company increases its output quantity by one unit. When calculating the cost norms in the DEA model for each individual company, weights are chosen such that the efficiency of the company is made as high as possible, given some restrictions[7]. In general, it will tend to be beneficial for a company to choose high weights for outputs of which it has relatively much, and low weights for other outputs. Figure 2.8 shows that the variation in observed weights among the companies is indeed very large. For example, the average weight per customer in 2006 was NOK 510, less than 1/5 of the maximum weight! We also see that many of the weights are equal to zero, which is related to the existence of slack. A company with a weight of zero for a particular output will normally have slack with respect to that output, i.e. the company produces less than the reference company. Thus, it is possible to produce more of the output (than the reference company does) without changing the total (minimized) cost, and slack can be interpreted as a "hidden" inefficiency, in the sense that it is not measured by the efficiency score of the company.

[6] In the EMS software used by NVE, the output weights are normalized, and can be interpreted as the marginal effect of an output increase on the company's efficiency score.
[7] See the mathematical formulation in Section 3.

	Average (NOK)		Max (NOK)		No. of zeros	
	2005	2006	2005	2006	2005	2006
Energy	21	32	93	92	68	48
Customers	605	510	2 343	2 671	73	82
Cottage customers	1 531	1 165	7 848	7 264	67	69
HV-lines	4 864	8 735	32 457	44 683	88	63
Net stations	15 979	12 896	45 769	52 548	50	59
Interface	1 174	1 300	7 032	7 701	69	51
Forest	29 284	28 184	222 056	215 491	44	57
Snow	18 445	24 193	109 824	123 595	73	58
Coast	22 847	22 700	148 469	165 919	82	81

Figure 2.8: Output weights (shadow prices) for 2005 and 2006

Relationship between prices

Comparing shadow prices on one output to the shadow prices on a different output, we notice also that the relative prices vary a lot. Figure 2.9 illustrates this for the Forest variable and the output variable High Voltage (HV) lines. Each point represents a company, and for some companies Forest has the highest price, while for others it is the HV variable. Moreover, many companies have a shadow price of zero for at least one of the outputs, indicating slack.

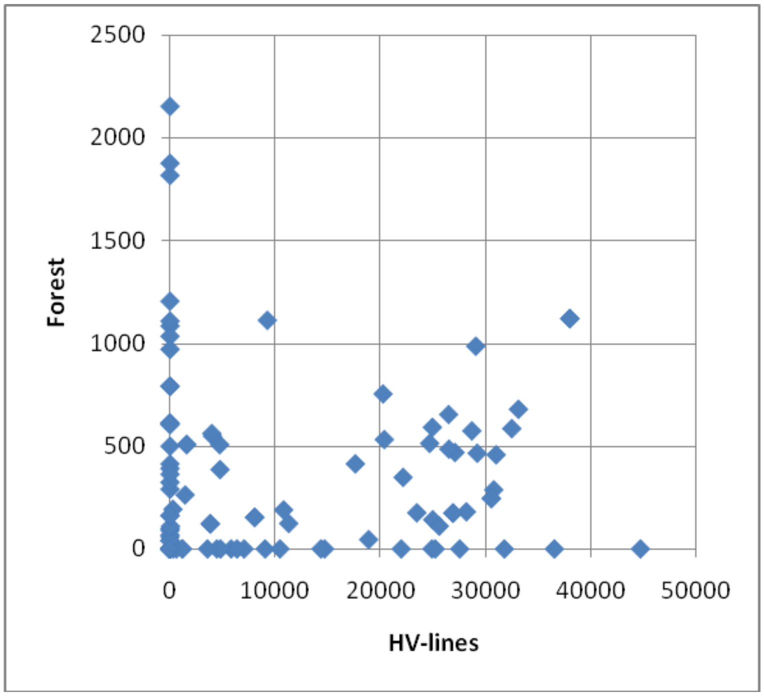

Figure 2.9: Shadow prices on Forest and High Voltage lines

Virtual weights

In order to assess the relative impact of the different outputs on the cost norms, we note that the cost norm of a company can be obtained by multiplying all its output quantities with the corresponding shadow prices or weights, and summing over the outputs. An example for Trollfjord Kraft, based on 2006-data, is shown in figure 2.10. There are four outputs with positive weights, and the five other ones have positive slack and zero weights. Note that Trollfjord Kraft has nothing of the interface output, so even though this output has a positive weight, it has no influence on the cost norm of Trollfjord Kraft. The total cost norm is 31.97 MNOK, and since the reported cost is 26.58 MNOK, the efficiency score of this company will be 120.3 % (= 31.97 / 26.58). Coast and energy are the most important output parameters for the company, making up 43.8 % and 42.5 % of the cost norm, respectively, while snow accounts for the remaining 13.7 %. The product of an output quantity and its weight is sometimes referred to as the virtual output quantity, and the corresponding percentage weight is called the virtual output weight, see Thanassoulis et al. (1987).

	Physical quantity	Slack	Weight (NOK)	Cost norm (1000 NOK)	Share of cost norm
Energy	147 367.0		92.1	13 580	42.5 %
Customers	4 670.0	596.1			
Cottage customers	494.0	431.3			
HV-lines	348.0	68.6			
Net stations	287.0	41.4			
Interface	0.0		905.4	0	0.0 %
Forest	101.6	512.0			
Snow	136 382.9		32.1	4 375	13.7 %
Coast	22.3		627 772.6	14 019	43.8 %
Sum				31 973	100.0 %

Figure 2.10: Computation of cost norm for Trollfjord Kraft (2006)

In figure 2.11 we show the composition of the cost norm for all the companies in the industry. Each column in the figure corresponds to one company, and since the width of the column is equal to the cost norm for the company, the area of the entire graph is equal to the total cost norm for the industry. The virtual output weights for the industry are given in brackets, and we see that energy and customers together constitute 59 % of the total cost norm for the industry. The geography variables, on the other hand, account for only 10 % of the norm, which may not seem very dramatic.[8] However, some companies have very high virtual weights for these three variables, as the 10 % are distributed on many small companies that represent a relatively small share of the total industry cost, but with large individual virtual weights.

[8] This does not mean that the industry cost norm increases by 10 %-points when the geography variables are introduced. As shown in figure 2.3 and 2.7, many companies are affected, but average efficiency in the industry increases from 89.8 % to 93.6 %, i.e. the cost norm increases by 3.8 %-points.

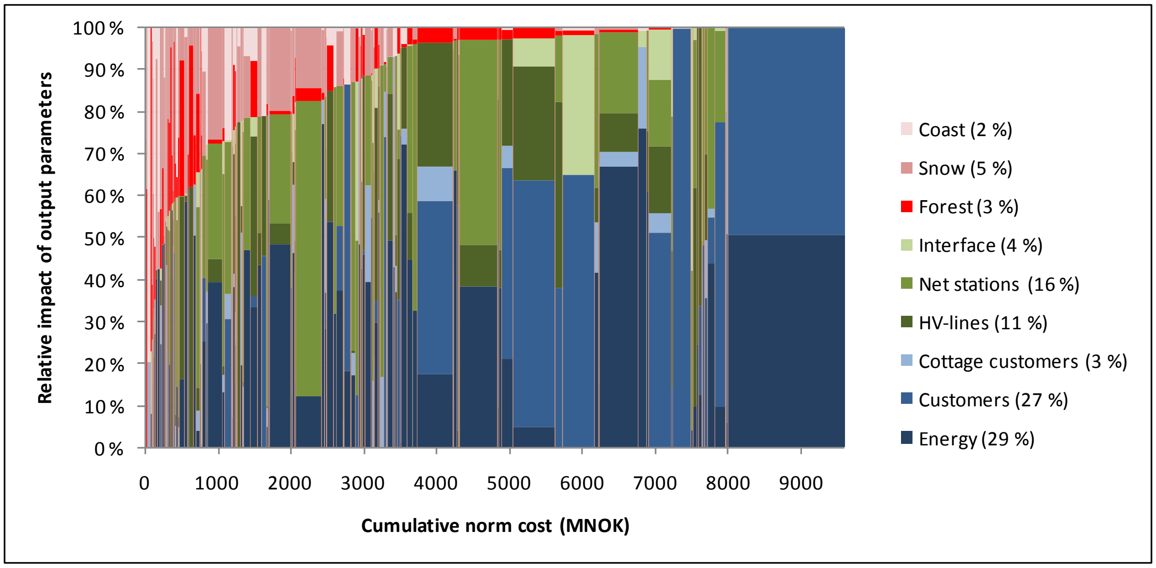

Figure 2.11: Virtual output weights for 2006

Figure 2.12 show companies with a virtual weight of more than 50 % on the geography variables. There are 13 such companies, and we see that some of them are highly super efficient (efficiency scores are shown in brackets). For these 13 companies, more than half of the cost norm will be determined by the geography variables. Although it is clearly difficult to distinguish between reasonable and unreasonable weights, we think that many will agree that the examples shown in figure 2.12 are unreasonable. Figure 2.13 shows companies with a weight of less than 10 % on energy and customers. There are 21 such companies, and again we see that some of these have very high efficiency scores.

In the 2006 data, 31 companies are super efficient, and 12 of them are represented in figures 2.12 and / or 2.13. Similarly, figure 2.14 shows all companies with efficiency scores of at least 110 % in 2006. There are 16 such companies, and 8 of these can also be found in figure 2.12 and/or figure 2.13, i.e., companies with extreme weights seem to be over-represented in the group of highly super efficient companies. This points towards a link between very high efficiency scores and extreme output weights, and this tendency is confirmed in figures 2.15 and 2.16, which show the relationships between virtual weights on geography variables and energy / customers on the one hand, and efficiency scores on the other hand. We notice that a larger virtual weight on geography variables tends to give higher efficiency scores, while it is the opposite for companies with large weight on the energy and customer variables.

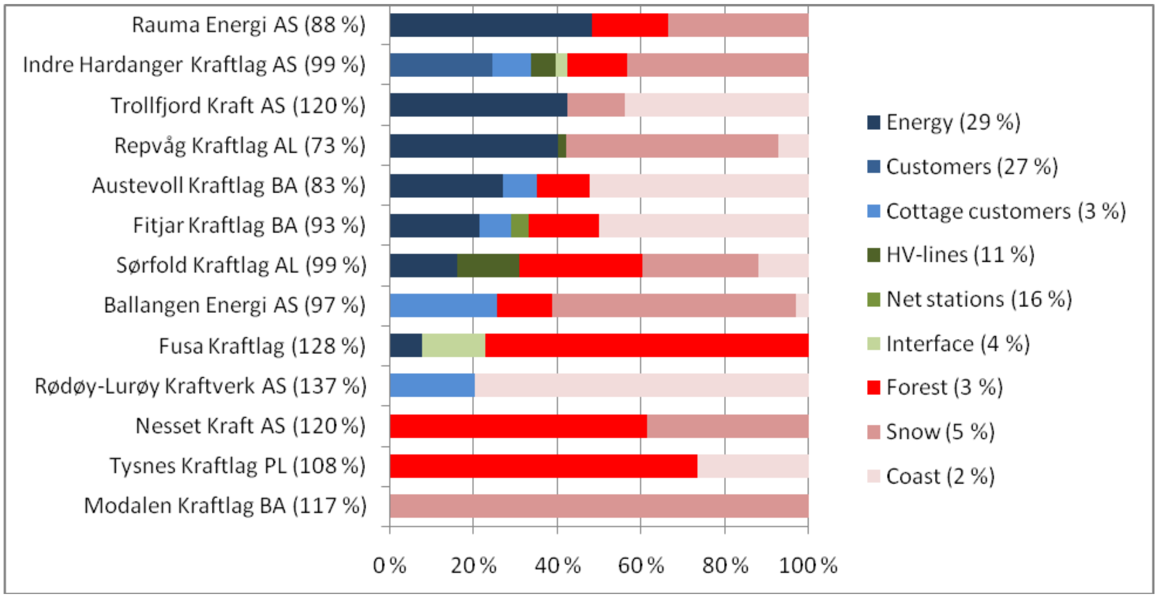

Figure 2.12: Companies with more than 50 % weight on geography (2006)

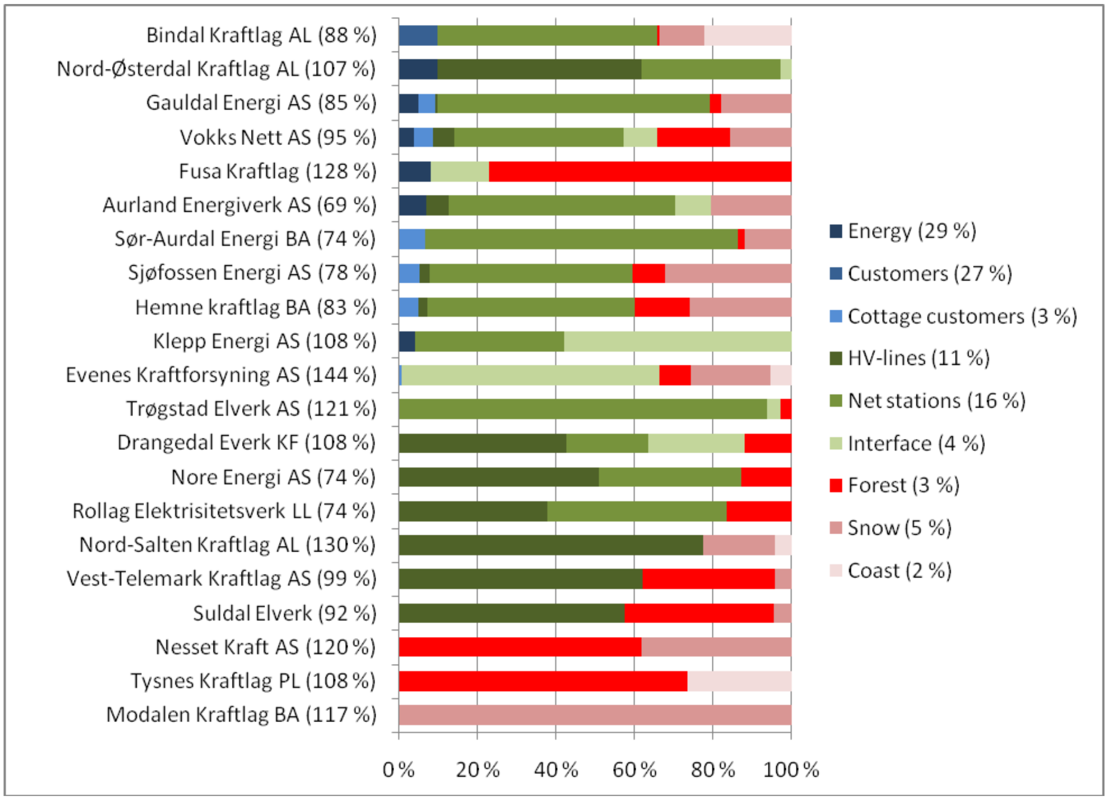

Figure 2.13: Companies with less than 10 % weight on energy/customers (2006)

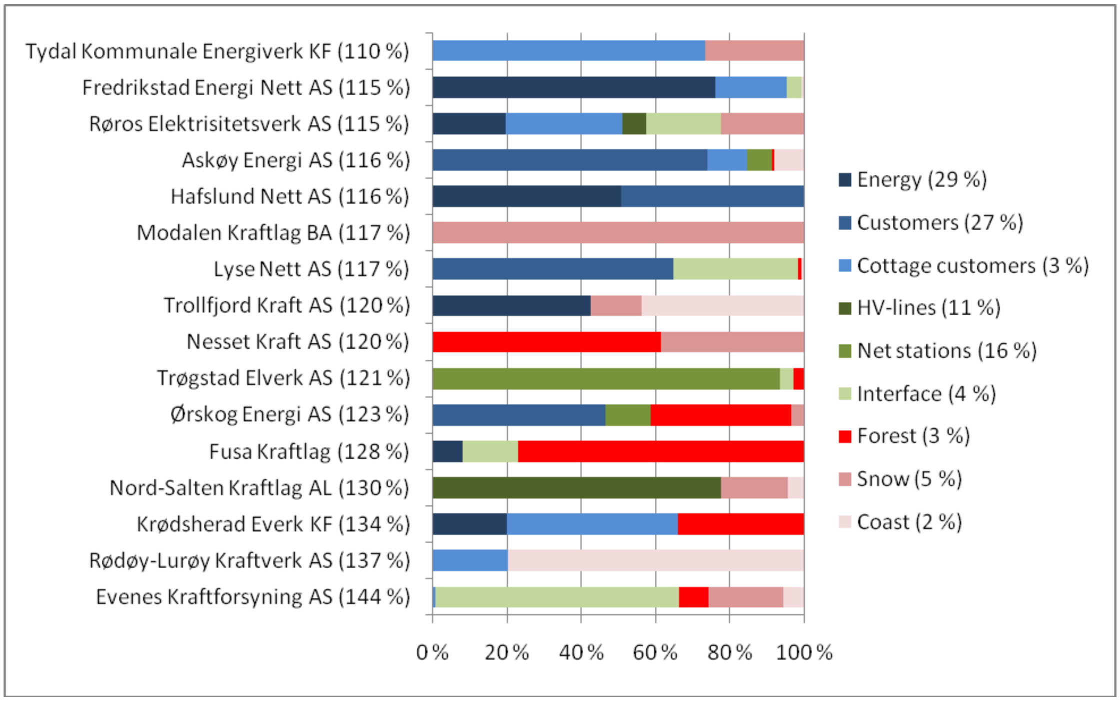

Figure 2.14: Companies with efficiency scores of more than 110 % (2006)

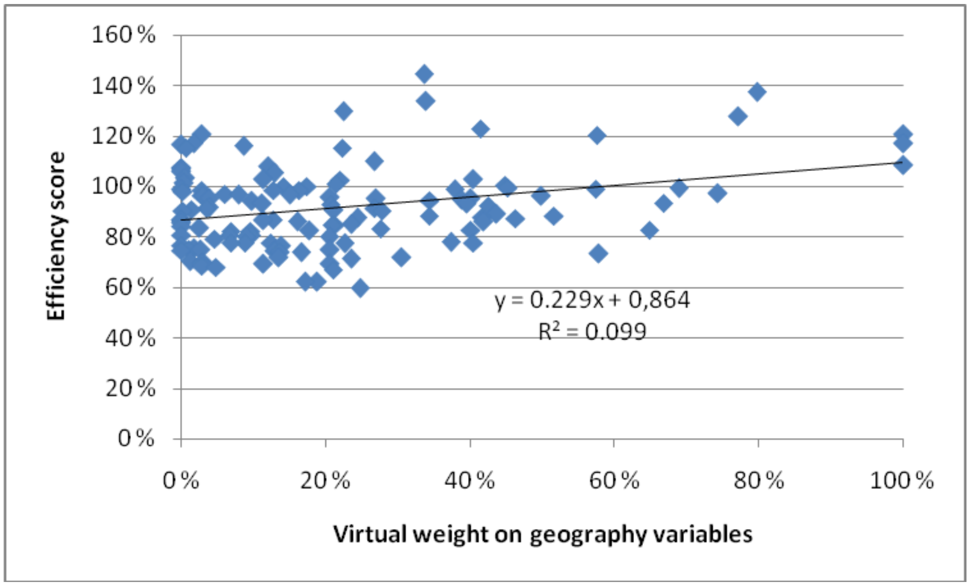

Figure 2.15: Efficiency and geography variables (2006)

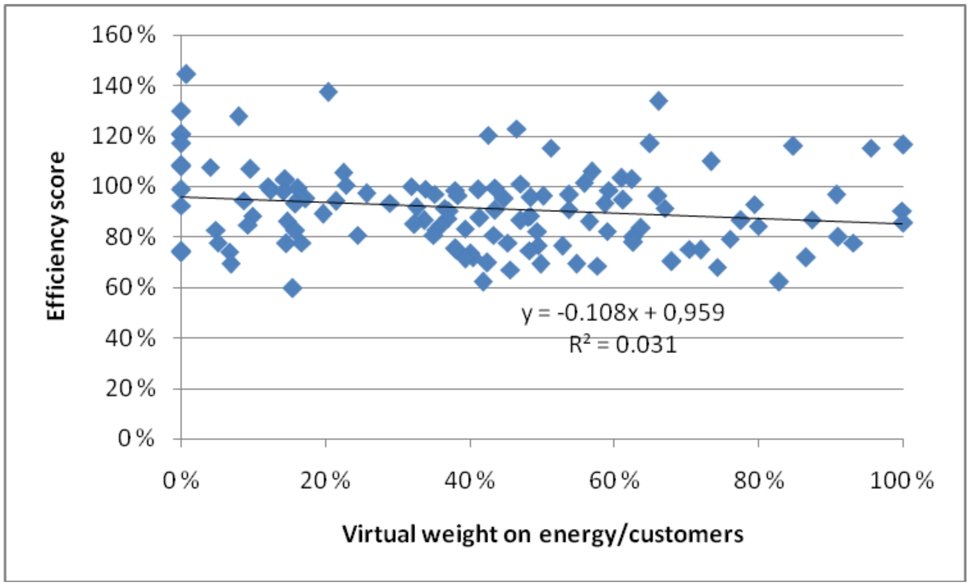

Figure 2.16: Efficiency and energy/customers (2006)

2.4 Summary

The examples in figures 2.11-2.16 show that extreme weights may be a problem in the present DEA model for the distribution companies, and that this phenomenon to a certain degree may explain the occurrence of very high efficiency scores. In the following sections, we look at possible remedies for this problem. In the DEA literature there exist several methods for handling the problem of extreme input/output weights. The most prominent method is to impose restrictions on the weights or shadow prices in the DEA model, and in the following we concentrate on different types of weight restrictions. However, other methods do exist, such as adjusting the data set by adding artificial data points, or adjusting DEA efficiency scores for slack. An overview of the different methods can be found in Thanassoulis (2004).

3. Weight restrictions

Generally, in DEA, the efficiency of a company is defined as the ratio of a weighted sum of outputs to a weighted sum of inputs. When computing the efficiency, there is complete freedom to choose the weights associated with each input and/or output so as to maximize the relative efficiency of the company. This complete flexibility in the selection of weights is especially important for identifying inefficient companies, as the DEA formulation demonstrates that these companies cannot achieve the maximum efficiency score even when they can choose the weights that show them in the best possible light.

However, the complete flexibility may result in some inputs and/or outputs being assigned a zero or negligible weight, meaning that these factors are in fact ignored in the efficiency assessment. Moreover, the weights may vary a lot from one company to another, and they may be in conflict with a priori beliefs about relative weights or rates of substitution. One way to limit the range of values that the weights can take is to use weight restrictions. Literature reviews on the use of weight restrictions in DEA can be found in Allen et al. (1997) and Thanassoulis et al. (2004).

Several types of weight restrictions have been proposed in the DEA literature. In this section we explain different versions of weight restrictions, and their interpretation in the DEA modeling framework. Our starting point is the DEA model specified for Norwegian distribution networks, as outlined in the previous section. Thus, our focus is on weight restrictions that fit into a cost efficiency model with a single input, total cost, and a number of outputs, consisting of product characteristics, like energy transported and the number of customers served, and environmental/geography variables, to account for the difficulty of providing network services in different concession areas[9]. The resulting DEA model for evaluating a specific company can be formulated as a linear program, either with an objective function that minimizes the efficiency score, or one that minimizes cost, thus establishing the corresponding cost norm for the evaluated company. We formulate the min cost variant in the following, in order to obtain a dual formulation with weights / prices that can be interpreted in monetary units.

[9] See Dyson and Thanassoulis (1988) for a discussion of the single-input model.

A linear program for determining the cost norm of company j^* is:

(LP1) $$\underset{\lambda}{\mathrm{Min}} \sum_{j \neq j^*} \lambda_j x_j$$

s.t.

$$\sum_{j \neq j^*} \lambda_j y_{rj} \geq y_{rj^*} \qquad r = 1,\ldots,s$$

$$\lambda_j \geq 0 \qquad j = 1,\ldots,n$$

There are n companies producing s different outputs. The total cost of company j is x_j while company j produces y_{rj} units of output r. The variable λ_j is the weight of company j in the reference set of the evaluated company j^*. The model is CRS (with constant returns to scale, $\lambda_j \geq 0$) and we assume super efficiency (sum over j except j^*). The interpretation of the linear program is that in the performance evaluation of company j^* we find the reference company, as a linear combination of the other companies in the industry, with minimum cost, such that it produces at least as much of each output as the evaluated company.

Alternatively, we may formulate the dual problem of LP1:

(LP2) $$\underset{p}{\mathrm{Max}} \sum_{r} y_{rj^*} p_{rj^*}$$

s.t

$$\sum_{r} y_{rj} p_{rj^*} \leq x_j \qquad j \neq j^*$$

$$p_{rj^*} \geq 0$$

The decision variables are the prices p_{rj*} for each output of the evaluated company, and the linear program can be interpreted so as to find prices for company j^* that maximize revenue, and at the same time assure that none of the other companies exceed their total cost at these prices (they are within a budget limit). The prices p_{rj*} in problem LP2 are the shadow prices of the output constraints in LP1, and consequently, p_{rj*} gives the increase in minimum cost due to an increase in y_{rj*}, and is a local per unit cost of output r.

Except for the budget constraint and the non-negativity constraints in LP2, there is complete freedom in choosing the shadow prices in the dual problem LP2. This may result in prices that are in contradiction to prior views or additional information. It may for instance be that the prices of different outputs turn out to be illogical. With slack in the inequality constraints in LP1, the corresponding shadow prices in LP2 will be equal to zero, and as a consequence, the minimum cost can be determined more or less completely by the weights of only a few outputs.

One possible solution to problems with the values of the weights is to limit the values that the prices can take in LP2. There are different versions of such weight restrictions, both restrictions on absolute prices and on the relationships between prices are possible. Moreover, it is possible to introduce restrictions on the products of prices and quantities. In the following we will show how weight restrictions can be formulated in the primal and dual LP problems of a benchmarking model of the NVE type, and we will give interpretations of the restrictions that we put on the weights.

3.1 Absolute weight restrictions

Absolute weight restrictions are upper or lower bounds on the absolute values of the shadow prices in LP2. Let us consider absolute weight restrictions on the shadow price of output *k*, i.e.

$$C_k^{LO} \leq p_{kj^*} \leq C_k^{UP}$$

These restrictions can be included in the dual program LP2, to give the following, more restricted linear program:

(LP3)

$$\operatorname*{Max}_{p} \sum_{r} y_{rj^*} p_{rj^*}$$

s.t.

$$\sum_{r} y_{rj} p_{rj^*} \leq x_j \qquad j \neq j^* \qquad (\lambda_j)$$

$$p_{kj^*} \leq C_k^{UP} \qquad (\mu_k^{UP})$$

$$-p_{kj^*} \leq -C_k^{LO} \qquad (\mu_k^{LO})$$

$$p_{rj^*} \geq 0$$

Introducing more restrictions into the maximum problem, the weight restrictions on the shadow prices of output *k* will have a non-positive effect on the optimal objective function value, i.e. the cost norm will be reduced (or stay the same), and the weight restrictions lead to stronger efficiency requirements. This is intuitive when we look at the effect on the corresponding primal cost minimization problem (taking the dual of LP3), with decision variables λ_j, μ_k^{UP}, and μ_k^{LO}, the latter two being the shadow prices of the added weight restrictions in LP3.

(LP4) $$\underset{\lambda,\mu}{\mathrm{Min}} \sum_{j \neq j^*} \lambda_j x_j + \mu_k^{UP} C_k^{UP} - \mu_k^{LO} C_k^{LO}$$

s.t.

$$\sum_{j \neq j^*} \lambda_j y_{kj} + \mu_k^{UP} - \mu_k^{LO} \geq y_{kj^*}$$

$$\sum_{j \neq j^*} \lambda_j y_{rj} \geq y_{rj^*} \qquad r \neq k$$

$$\lambda_j, \mu_k^{UP}, \mu_k^{LO} \geq 0$$

From LP4 we see that an interpretation of the restrictions on the absolute value of the shadow prices of output *k*, is that it is possible to buy and sell output *k* at prices C_k^{UP} and C_k^{LO}, respectively. In other words, the peers of the evaluated company can either produce output *k* itself or engage in an external market, buying μ_k^{UP} and selling μ_k^{LO} at prespecified prices C_k^{UP} and C_k^{LO}. This can also be interpreted as introducing another peer (the external market) that can provide output *k* at price C_k^{UP} per unit and take care of any surplus at price C_k^{LO} per unit.

3.2 Relative weight restrictions

Relative weight restrictions limit the relationship between shadow prices of different outputs. Let us consider relative weight restrictions on the shadow prices of outputs *l* and *m*, i.e.

$$C_{lm}^{LO} p_{mj^*} \leq p_{lj^*} \leq C_{lm}^{UP} p_{mj^*}$$

These restrictions can be included in the dual program LP2, to give the following, more restricted linear program:

(LP5)
$$\underset{p}{\text{Max}} \sum_r y_{rj^*} p_{rj^*}$$

s.t.

$$\sum_r y_{rj} p_{rj^*} \leq x_j \qquad j \neq j^* \qquad (\lambda_j)$$

$$p_{lj^*} - C_{lm}^{UP} p_{mj^*} \leq 0 \qquad (\gamma_{lm}^{UP})$$

$$-p_{lj^*} + C_{lm}^{LO} p_{mj^*} \leq 0 \qquad (\gamma_{lm}^{LO})$$

$$p_{rj^*} \geq 0$$

Once more, introducing more restrictions into the maximum problem, the weight restrictions on the shadow prices of outputs *l* and *m* will have a non-positive effect on the optimal objective function value, i.e. the cost norm will be reduced (or stay the same), and the weight restrictions lead to stronger efficiency requirements. This is intuitive when we look at the effect on the corresponding primal cost minimization problem (taking the dual of LP5), with decision variables λ_j, γ_{lm}^{UP}, and γ_{lm}^{LO}, the latter two being the shadow prices of the added weight restrictions in LP5.

(LP6)
$$\underset{\lambda,\gamma}{\text{Min}} \sum_{j \neq j^*} \lambda_j x_j$$

s.t.

$$\sum_{j \neq j^*} \lambda_j y_{lj} + \gamma_{lm}^{UP} - \gamma_{lm}^{LO} \geq y_{lj^*}$$

$$\sum_{j \neq j^*} \lambda_j y_{mj} - C_{lm}^{UP} \gamma_{lm}^{UP} + C_{lm}^{LO} \gamma_{lm}^{LO} \geq y_{mj^*}$$

$$\sum_{j \neq j^*} \lambda_j y_{rj} \geq y_{rj^*}, \; r \neq l, m$$

$$\lambda_j, \gamma_{lm}^{UP}, \gamma_{lm}^{LO} \geq 0$$

An interpretation of the relative weight restrictions is that additional to the production by the reference companies, it is possible to substitute outputs *l* and *m* in fixed proportions, given by C^{UP} and C^{LO}. It is for instance possible to obtain a unit of output *l* by giving up C_{lm}^{UP} units of output *m*.

3.3 Virtual weight restrictions

Virtual weight restrictions limit the value of the virtuals, i.e. the product of the output variable and its shadow price. Let us consider a virtual weight restriction on output *k*. This takes the form of restricting the share that output *k* contributes to the total cost norm of company j^* in the objective function of the linear program LP2:

$$C_k^{LO}\sum_r p_{rj^*}y_{rj^*} \le p_{kj^*}y_{kj^*} \le C_k^{UP}\sum_r p_{rj^*}y_{rj^*}, \quad 0 \le C_k^{LO} \le C_k^{UP} \le 1$$

These restrictions can be included in the dual program LP2, to give the following, more restricted linear program:

(LP7)

$$\operatorname*{Max}_p \sum_r y_{rj^*} p_{rj^*}$$

s.t.

$$\sum_r y_{rj} p_{rj^*} \le x_j \qquad j \ne j^* \qquad (\lambda_j)$$

$$p_{kj^*}y_{kj^*} - C_k^{UP}\sum_r p_{rj^*}y_{rj^*} \le 0 \qquad (\rho_k^{UP})$$

$$-p_{kj^*}y_{kj^*} + C_k^{LO}\sum_r p_{rj^*}y_{rj^*} \le 0 \qquad (\rho_k^{LO})$$

$$p_{rj^*} \ge 0$$

Again, introducing more restrictions into the maximum problem, the virtual weight restrictions on output *k* will have a non-positive effect on the optimal objective function value, i.e. the cost norm will be reduced, and the weight restrictions lead to stronger efficiency requirements. Also in this case, it is possible to investigate the effect on the corresponding primal cost minimization problem by taking the dual of LP7, with decision variables λ_j, ρ_k^{UP}, and ρ_k^{LO}, the latter two being the shadow prices of the added virtual weight restrictions in LP7.

(LP8) $$\underset{\lambda,\rho}{\text{Minimer}} \sum_{j \neq j^*} \lambda_j x_j$$

s.t.

$$\sum_{j \neq j^*} \lambda_j y_{kj} + \rho_k^{UP}(1 - C_k^{UP}) y_{kj^*} - \rho_k^{LO}(1 - C_k^{LO}) y_{kj^*} \geq y_{kj^*}$$

$$\sum_{j \neq j^*} \lambda_j y_{rj} - \rho_k^{UP} C_k^{UP} y_{rj^*} + \rho_k^{LO} C_k^{LO} y_{rj^*} \qquad \geq y_{rj^*}, r \neq k$$

$$\lambda_j, \rho_k^{UP}, \rho_k^{LO} \geq 0$$

Also in this case, the effect is some sort of substitution possibility that is introduced in the cost minimization, giving new feasible solutions, and thus having a non-positive effect on the value of the objective function, compared to the unrestricted LP1. It is also possible to restrict not only the virtual of a single output, but the combined effect on the objective function of several outputs. This is discussed further in section 5.

3.4 Summary

In the DEA literature a variety of different restrictions on shadow prices / weights are described. For the DEA model that NVE is using for distribution networks, with a single input equal to total cost, and various outputs, the most relevant weight restrictions are absolute and relative weight restrictions, restricting the absolute values or relative values of shadow prices, as well as virtual weight restrictions, restricting the effect that one or a combination of outputs can have on the cost norm. Relative weight restrictions will be considered in section 4, where we evaluate and revise some of the proposed restrictions in NVE (2008), while an alternative approach based on virtual weight restrictions is proposed and evaluated in section 5. Restrictions with respect to absolute levels of the weights do not seem natural in the case of the geography variables, and will not be considered in this report. They may be useful in the case of other variables, such as delivered energy and customers served, and restrictions on these variables will be the subject of a later report.

4. Weight restrictions proposed by NVE

In this section we will discuss some of the relative weight restrictions proposed by NVE (2008). The entire proposal is presented in section 4.1, together with a brief discussion of the motivation for the different restrictions, as stated in the NVE report. In section 4.2 we evaluate the restrictions with respect to the geography variables, and we suggest a revised formulation of these. Among the changes that we propose is a redefinition of the geography variables in order to make the variables and their weights more easily interpretable, thereby facilitating the formulation of weight restrictions. Then, in section 4.3 we evaluate the effects of the revised restrictions, and section 4.4 gives a summary and conclusions.

4.1 The proposal

The restrictions proposed in the NVE report are shown in figure 4.1 below. They are all of the relative type, and are based on pair-wise comparisons of output weights. Restrictions VR1-VR8 are two-sided, thereby providing both upper and lower bounds for the involved weights, while restrictions VR9-VR11 are one-sided, and form an upper bound for the geography weights based on the weight of HV-lines.

Restriction(s)	Involved variables	Mathematical formulation
VR1 / VR2	HV-lines versus net stations	$0.952 p_{NS} \leq p_{HV} \leq 8.572 p_{NS}$
VR3 / VR4	Interface versus net stations	$0.02304 p_{NS} \leq p_{Int} \leq 0.20738 p_{NS}$
VR5 / VR6	Customers versus cottage customers	$1/3 p_{Cust} \leq p_{CCust} \leq 3 p_{Cust}$
VR7 / VR8	Net stations versus customers	$1.618 p_{Cust} \leq p_{NS} \leq 58.252 p_{Cust}$
VR9	Forest versus HV-lines	$p_{Forest} \leq 0.04 p_{HV}$
VR10	Snow versus HV-lines	$p_{Snow} \leq 0.0053 p_{HV}$
VR11	Coast versus HV-lines	$p_{Coast} \leq 36.364 p_{HV}$

Figure 4.1: Weight restrictions in NVE (2008)

The various restrictions have different motivations. According to the report, restrictions VR1-VR8 are introduced in order to reduce slack in the DEA-model, while the motivation behind VR9-VR11 is to avoid unreasonably high efficiency scores as a result of the geography variables. As we discussed in section 2, high efficiency scores may in some cases reflect extreme weighting of outputs rather than real efficiency, and weight restrictions can clearly be used to eliminate such weighting schemes. Hence, the motivation behind VR9-VR11 seems plausible. We find the motivation behind VR1-VR8, i.e., to reduce slack, somewhat more problematic. It is indeed true that slack in a DEA analysis represents a form of “hidden” inefficiency. By choosing zero weights for some outputs, companies may be able to weight their “preferred” outputs more heavily, thereby obtaining higher efficiency scores. In this sense, the existence of slack is connected to the problem of “unreasonable” efficiency scores. However, eliminating slack does not in itself solve the problem of unreasonable weighting schemes / efficiency scores. Note that, in order to eliminate slack for an output, it is enough to force the corresponding weight to be strictly positive. However, the resulting weight may still be very low relative to other output weights, and may be seen as highly unreasonable. Hence, in order to evaluate the DEA weights of a particular company, it is not enough to check whether the values of the weights (slacks) are positive or not, one needs to look at the actual values of the various weights and conclude whether they represent a plausible weighting scheme or not. An interesting example of such an evaluation can be found in Thanassoulis et al. (1987), who introduce the concept of “well-rounded performance” as an additional check on a company’s efficiency score, meaning that the efficiency rating “is based fairly evenly on all its outputs and inputs”.

Another concern with respect to VR1-VR8 in the DEA model for distribution networks is that some of the output variables are input factors, like for instance HV-lines, net stations, and interface. For these variables, it can even be argued that the existence of slack should be seen as positive, since it indicates that the evaluated company uses less of an input than the reference company. For these outputs, it is not obvious that one should seek to reduce slack!

4.2 Evaluation and reformulation of the geography restrictions

The restrictions VR9-VR11, shown in figure 4.1 above, relate the geography weights to the weight on HV-lines. The intention behind them is to limit the weight of each one of the geography variables to twice the weight of HV-lines. As stated in section 4.1, we find the motivation behind these restrictions plausible, but we have some objections to the details of their formulation, and this has mainly to do with the scaling factors that are applied to the left and right hand sides of the restrictions in order to make the geography weights and the weight of HV-lines comparable. We argue that some errors are introduced via this scaling procedure, and we therefore propose an alternative formulation. We scale the output *quantities* of the DEA model in order to make them comparable, thereby avoiding the scaling of output *weights*. A positive side effect of our modified proposal is that the output quantities for the geography variables become easier to interpret, thereby making the DEA model more understandable.

Restrictions VR9-VR11 in NVE (2008) are formulated as one-sided restrictions, whereby an upper limit for the geography weights are specified relative to the weight on HV-lines. An interesting question is whether one should also specify lower limits for these weights, since some companies may be able to obtain unreasonably high efficiency scores by assigning very low weights to the geography variables. In the following, however, we will limit the discussion to the restrictions proposed by NVE (2008).

Forest versus HV-lines (VR9)

The forest variable of company *j* is defined as

$$y_{Forest,j} = ForestIndex_j \cdot HighVoltageLinesAir_j \, , \qquad (4.1)$$

where the forest index measures the share of the company's area that is covered by high-growth forest (0-100), and HV-lines are measured in no. of kilometers. Based on this variable definition, NVE (2008) proposes the weight restriction

$$p_{Forest} 100/2 \leq 2 p_{HV} \, , \qquad (4.2)$$

which is equivalent to

$$p_{Forest} \leq 0.04 p_{HV} \, . \qquad (4.3)$$

The weight of HV-lines on the right hand side of (4.2) is multiplied by 2 in order to limit the weight of the forest variable to at most two times the weight of HV-lines. The factor

100 on the left hand side is introduced in order to adjust for the fact that forest index values are numbers between 0-100, and the division by 2 is made because air cables account for roughly 50 % of the high voltage network in Norway. We do not agree with the latter adjustment, since output weights represent marginal values. The marginal value of the last kilometer of "forest line" should be compared to the marginal value of the last kilometer of "normal" HV-line, and it is therefore wrong to adjust the weights based on the average composition of the network. Since the division by 2 on the left hand side is equivalent to multiplying by 2 on the right hand side, the proposed restriction is indeed much weaker than what was intended.

In order to simplify the restriction, we propose instead to rescale the forest variable in the following manner:

$$y_{Forest,j} := ForestIndex_j / 100 \cdot HighVoltageLinesAir_j \, . \tag{4.4}$$

By dividing by 100, the forest index can be interpreted as the fraction of the company's area with high-growth forest, and hence the redefined variable can be interpreted as the number of kilometers of lines exposed to high-growth forest. Hence, the forest variable will have the same unit of measurement as the HV-line variable, and the weight restriction can be simplified to:

$$p_{Forest} \le 2 p_{HV} \, . \tag{4.5}$$

Snow versus HV-lines (VR10)

The snow variable of company *j* is defined as

$$y_{Snow,j} = SnowIndex_j \cdot HighVoltageLinesAir_j \, , \tag{4.6}$$

where the snow index measures the average precipitation as snow (in millimeters per year). The weight restriction in NVE (2008) is formulated as

$$p_{Snow} 757 / 2 \le 2 p_{HV} \, , \tag{4.7}$$

which is equivalent to

$$p_{Snow} \leq 0.0053 p_{HV} . \tag{4.8}$$

The weight of HV-lines on the right hand side of (4.7) is multiplied by 2 in order to limit the snow weight to two times the value of the weight on HV-lines. The number 757 on the left hand side is the maximum amount of snow precipitation, where the precipitation number has been adjusted by multiplying it by the proportion of air cables in the company's high voltage network. The division by 2 on the left hand side is made for the same reason as in (4.2), i.e. because air cables account for 50 % of the Norwegian high voltage network. We believe this adjustment should be rejected for the same reason as in the case of the forest variable.

We propose a similar reformulation of the snow variable as in the case of the forest variable. The snow index is rescaled to a number between 0 and 1, by dividing by the maximum observed value (and without correcting for the proportion of air cables in the company's network):

$$y_{Snow,j} := \frac{SnowIndex_j}{SnowIndex_{MAX}} \cdot HighVoltageLinesAir_j \tag{4.9}$$

The new snow variable can be interpreted as the number of kilometers of maximally snow exposed HV-lines. Given the re-definition of the snow variable, the corresponding weight restriction can be written as

$$p_{Snow} \leq 2 p_{HV} , \tag{4.10}$$

assuming that we want to use the restriction ratio equal to 2, proposed by NVE (2008).

Coast versus HV-lines (VR11)

The coast variable of company *j* is defined as

$$y_{Coast,j} = CoastIndex_j \cdot HighVoltageLinesAir_j , \tag{4.11}$$

where the coast index is defined as average wind speed divided by average distance to coast. The corresponding weight restriction in NVE (2008) is formulated as

$$p_{Coast} 0.11 / 2 \leq 2 p_{HV}, \tag{4.12}$$

which is equivalent to

$$p_{Coast} \leq 36.364 p_{HV}. \tag{4.13}$$

The weight of HV-lines on the right hand side of (4.12) is multiplied by 2 in order to limit the coast weight to two times the value of the weight on HV-lines. The number 0.11 on the left hand side is the maximum amount of the coast index, where the index has been adjusted by multiplying it by the proportion of air cables in the company's high voltage network. The division by 2 on the left hand side is made for the same reason as for the forest and snow weight restrictions in (4.2) and (4.7), and should be rejected for the same reasons as explained earlier.

We rescale the coast variable in a similar manner as for the snow variable in (4.9), by defining[10]

$$y_{Coast,j} := \frac{CoastIndex_j}{CoastIndex_{MAX}} \cdot HighVoltageLinesAir_j, \tag{4.14}$$

which can be interpreted as the number of kilometers of maximally exposed, with respect to coastal factors, HV-lines. The corresponding weight restriction then becomes

$$p_{Coast} \leq 2 p_{HV}. \tag{4.15}$$

Effect of reformulation

Note that the rescaling of the output variables, as defined by (4.4), (4.9) and (4.14), does not in itself change the DEA results. This is illustrated by figure 4.2 below, where we compare the efficiency scores based on the original data set (horizontal axis) and the corresponding efficiency scores based on the rescaled geography variables (vertical axis). As we can see from the figure, the two formulations are equivalent. However, the output weights of the geography variables are affected, as illustrated by the table in figure 4.3.

[10] As for the snow variable, we have computed the maximum coast index value based on unadjusted index numbers, and not as in NVE (2008).

The average value of the forest weight has increased by a factor of 100, which is exactly the factor that we have used to scale the new forest variable in (4.4). The average value of the snow weight with the original data set was NOK 20.27, whereas the average value with the reformulated data set is NOK 24 139, i.e., the value has increased by a factor of 1193.6. This factor corresponds to the maximum observed value of the snow index[11], i.e., the value that we used to define the new snow variable in (4.9). The coast weight has increased by a factor of 0.1611, which corresponds to the maximum observed value of the coast index[12], i.e., the value used to define the new coast variable in (4.14).

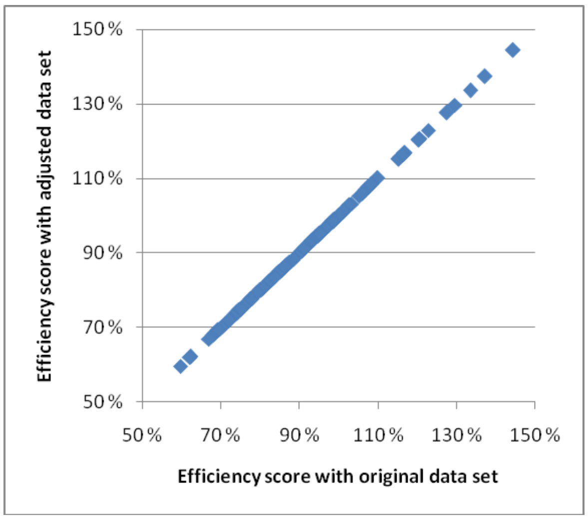

Figure 4.2: Effect of reformulation on efficiency scores – unrestricted model (2006)

[11] Observed index value for Odda Energi AS.
[12] Observed index value for Tafjord Kraftnett AS.

	Average (NOK)		Max (NOK)		No. of zeros	
	Old	New	Old	New	Old	New
Energy	32	32	92	92	48	48
Customers	510	510	2 671	2 671	82	82
Cottage customers	1 165	1 165	7 264	7 264	69	69
HV-lines	8 735	8 735	44 683	44 683	63	63
Net stations	12 896	12 896	52 548	52 548	59	59
Interface	1 300	1 300	7 701	7 701	51	51
Forest	282	28 184	2 155	215 491	57	57
Snow	20	24 193	104	123 595	58	58
Coast	140 948	22 700	1 030 215	165 919	81	81

Figure 4.3: Effect of reformulation on output weights (2006)

Figure 4.3 above shows that the average weight of HV-lines in 2006 is NOK 8 735, while the average weights of the geography variables are approximately 2.5 to 3 times as large. Hence, using a factor of 2 in the relative weight restrictions, as given by (4.5), (4.10) and (4.15), will clearly have an effect on the DEA results. This is also apparent from the diagrams in figure 4.4 below, where we have plotted the observed combinations of the unrestricted geography weights (vertical axes) and HV weights (horizontal axes). The solid lines in the diagrams indicate the relative weight restrictions given by (4.5), (4.10) and (4.15), and we see that a large number of the observed combinations of weights violates the proposed restrictions. We would therefore expect the restrictions to affect a large number of companies.

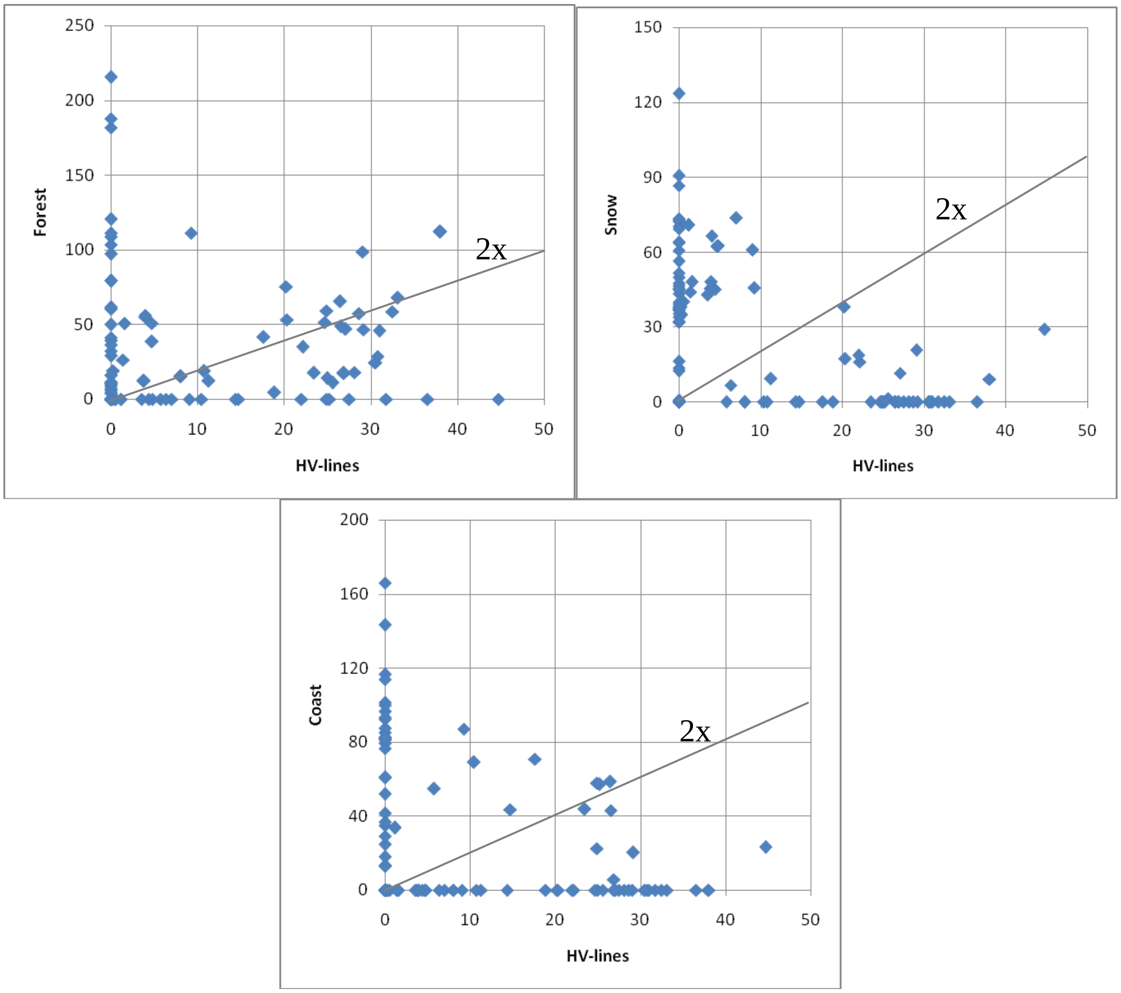

Figure 4.4: Unrestricted output weights with rescaled data (2006**)**

4.3 Effects of relative restrictions

Figure 4.5, where we have plotted the efficiency scores corresponding to the original and revised NVE proposal, as well as the unrestricted efficiency scores, illustrates that the revised version of the NVE restrictions will indeed affect a large number of companies, as was expected. Figure 4.6 gives a list of the 15 companies for which the efficiency score is reduced most, and we see that for the companies that have a reduction in efficiency of at least 10 %-points, most companies, with two exceptions, would have been evaluated as (very) super-efficient given the unrestricted model.

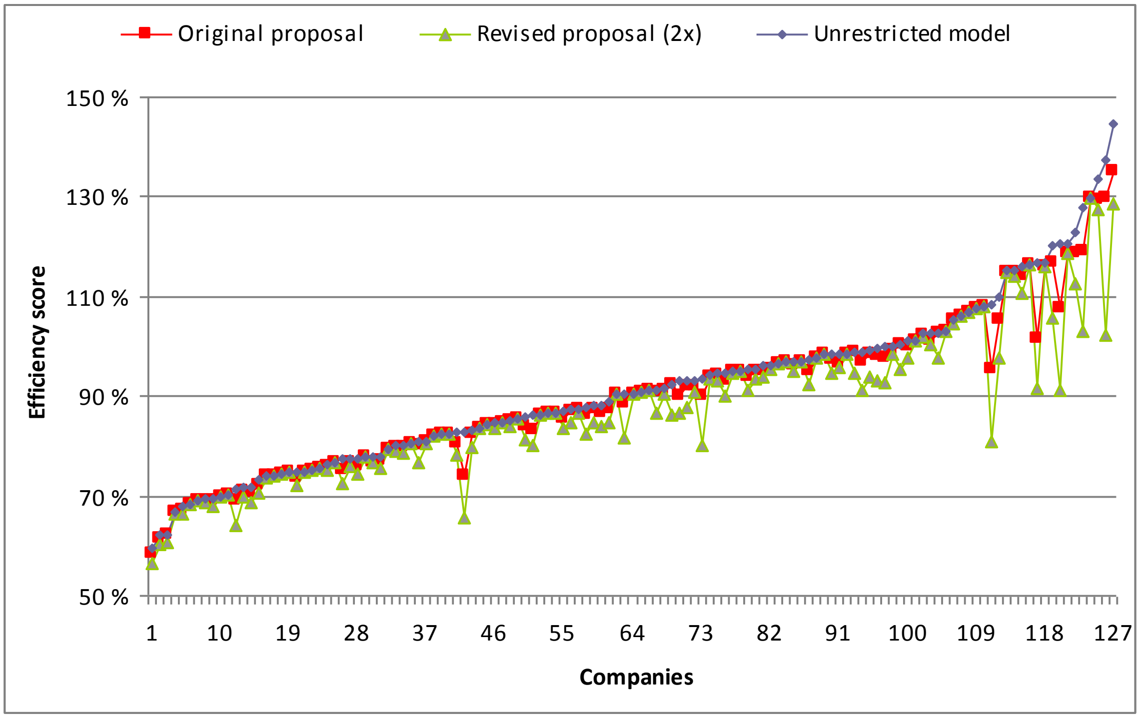

Figure 4.5: Effect of original and revised proposal (2006)

Company	Unrestricted	Restricted	Reduction
Rødøy-Lurøy Kraftverk AS	137	102	35
Nesset Kraft AS	120	91	29
Tysnes Kraftlag PL	108	81	28
Modalen Kraftlag BA	117	91	25
Fusa Kraftlag	128	103	25
Austevoll Kraftlag BA	83	66	17
Evenes Kraftforsyning AS	144	129	16
Trollfjord Kraft AS	120	106	15
Fitjar Kraftlag BA	93	80	13
Tydal Kommunale Energiverk KF	110	98	12
Ørskog Energi AS	123	112	10
Finnås Kraftlag	90	82	9
Indre Hardanger Kraftlag AS	99	91	8
Nordvest Nett AS	100	93	8
Fjelberg Kraftlag	71	64	7

Figure 4.6: The 15 most affected companies, based on efficiency scores (2006)

Figure 4.5 also illustrates another interesting point, namely, that the revised version of the restrictions in the NVE proposal has a stronger effect on the efficiency scores than the original version. This is due to the adjustment made by NVE (2008) to account for the fact that air cables only account for 50 % of the high voltage network. This adjustment is based on an erroneous argument, and has therefore been removed in our revised version. Since the adjustment was made by dividing the left hand side of the restrictions (4.2), (4.7) and (4.12) by 2, its effect was to weaken the restrictions, and removing it will therefore result in stronger restrictions. Figure 4.7 compares the effect of the original restrictions proposed by NVE (horizontal axis) to restrictions on the reformulated data set (vertical axis). With the reformulated data set, we have used a ratio of 4 in the restrictions, i.e., the geography weights are bounded upwards by 4 times the weight of HV-lines. We see that the efficiency scores for the two formulations are nearly[13] identical for most companies. Hence, the restrictions proposed by NVE roughly correspond to using a restriction ratio equal to 4 in the weight restrictions, i.e., they are in fact much weaker than what is stated in the proposal.

[13] The differences are due to the fact that the maximum values of the snow and coast indices, used to scale output weights in the NVE proposal and output quantities in the case of our reformulation, are not identical. In NVE (2008) the maximum values of the snow and coast indices are calculated with respect to adjusted index values, where the indices are multiplied by the proportion of air cables in the individual company networks. In our reformulation, we have not made any adjustments when calculating the maximum snow and coast index values, as we think this is neither necessary nor correct.

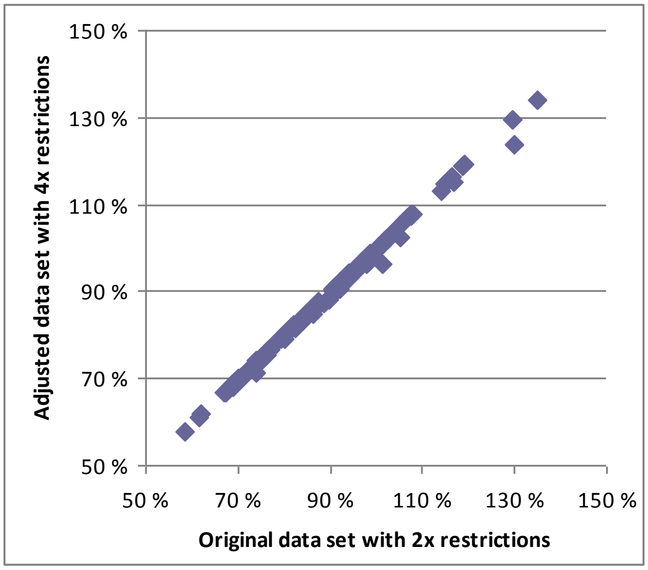

Figure 4.7: Effect of reformulation on efficiency scores – restricted model (2006)

The choice of the restriction ratio equal to 2 in our revised version of the NVE proposal is consistent with the original proposal. However, when making this choice, one needs to interpret the geography weights correctly. Note that HV-lines exposed to one of the geography factors are counted twice in the output data. Firstly, they are counted by the HV-variable, and secondly, by the relevant geography variable. Suppose, e.g., that we would like to limit the value of one kilometer of forest-exposed HV-line to twice the value of one kilometer of non-exposed HV-line. Considering the double counting of forest-exposed lines, the correct restriction with respect to the forest weight should be $p_{Forest} + p_{HV} \leq 2p_{HV}$, which is equivalent to

$$p_{Forest} \leq p_{HV}, \qquad (4.16)$$

i.e., corresponding to a ratio of 1 in the weight restriction. This is a considerably stronger requirement than the ratio of 4 implied by the original proposal in NVE (2008).

If one chooses to implement relative restrictions of this type, the final choice of this ratio should be based on further analyses of the companies' cost structure. The sensitivity analysis shown in figure 4.8 illustrates some of the effects that this choice may have. We have analyzed seven different cases, with a restriction ratio ranging from 0 to 5, as well as the unrestricted case. Note that a limit ratio of 0 is equivalent to the case where the geography variables are not included in the DEA model. We see that for all the analyzed cases, the average (cost-weighted) industry efficiency does not vary much. When the restriction ratio is increased, fewer companies will experience very large reductions, but the number of affected companies does not decrease dramatically. Even with a restriction ratio of 3, 81 of the 127 companies are affected, and the average reduction for these 81 companies will be 3.7 percentage points (using a simple average). Compared to the virtual weight restrictions that we shall look at in the next section, the efficiency reductions caused by the relative weight restrictions seem to be spread out over a fairly large number of companies.

		Maximum geography weights relative to HV-weight						
		0x	**1x**	**2x**	**3x**	**4x**	**5x**	**Unrestr.**
Industry efficiency		89.8 %	91.5 %	92.4 %	92.8 %	93.0 %	93.2 %	93.6 %
No. of affected comp.		115	103	91	81	79	78	0
Average reduction		9.7	6.9	5.0	3.7	2.7	2.3	-
Reduction in %-points	**Over 50**	2	1	0	0	0	0	0
	25 - 50	8	4	4	0	0	0	0
	10 - 25	29	15	7	7	6	3	0
	5 - 10	22	22	14	6	3	5	0
	0 - 5	54	61	66	68	70	70	0
	No change	12	24	36	46	48	49	127
Corr(Eff, VirtualGeography)		-	-0.15	-0.02	0.10	0.17	0.20	0.32
Corr(Eff, PhysicalGeography)		-0.43	-0.24	-0.10	0.00	0.05	0.07	0.16
Corr(Eff, VirtualProducts)		0.00	-0.04	-0.09	-0.13	-0.15	-0.15	-0.18

Figure 4.8: Sensitivity analysis w.r.t. choice of restriction ratio (2006)

In section 2 we showed that the unrestricted efficiency scores are positively correlated with the virtual weight on geography, and negatively correlated with the virtual weight on the product variables energy / customers. The correlation coefficents shown in the lower part of figure 4.8 illustrate that this correlation will be affected by the introduction of relative weight restrictions. The correlation between efficiency and the virtual geography weight decreases when the restrictions are tightened, and eventually becomes negative. Zero correlation occurs for a restriction ratio of between 2 and 3. Note that, since the virtual weights are optimized for each company in order to evaluate the company in the best possible light, we may overestimate the correlation effect by using virtual geography weights. Therefore we also show correlation coefficients between efficiency scores and the physical value of the geography variables[14], and this analysis shows a similar effect of the weight restrictions. Figure 4.8 also shows that the correlation between efficiency scores and the virtual weight on the product variables increases as the restrictions are tightened. Zero correlation in this case is obtained only when the geography variables are removed from the model, i.e., the restriction ratio is set equal to zero.

4.4 Summary and conclusions

In this section we have discussed the relative weight restrictions proposed in NVE (2008). We find the restrictions on the geography variables plausible and have suggested a reformulation, in section 4.2, of the data set in order to make the geography variables and their weights easier to interpret. With the revised data set, the geography weight restrictions can be formulated more easily. We showed, in section 4.3, that the revised restrictions are much stronger than the restrictions of the original proposal, due to an erroneous adjustment in the original restrictions. Finally, we performed some sensitivity analyses with respect to the restriction ratios, and one of the conclusions was that the relative restrictions seem to affect a large number of the companies in the industry, even when the ratios are high. The sensitivity analyses also show that the correlation effects discussed in section 2 will be influenced by the introduction of weight restrictions.

Note that we have not presented any evidence to support the choice of particular restriction ratios. In order to make such a choice, we need more information about the cost structure of the industry, and we need a better understanding of how the DEA model represents the cost norm via the output weights. Understanding what the output weights really mean may be the most serious challenge when implementing relative weight restrictions, and this is partly due to the fact that some cost drivers have been excluded from the model, thereby making it harder to interpret the weights of the respective cost drivers that *are* included. We will come back to this problem in the next section, and present a possible solution to it.

[14] For each company, a single geography measure is computed by taking the average of the three geography variables. In order to remove scale effects, the physical values have been divided by the number of kilometers of HV-lines for each company.

5. Alternative methods: Virtual weight restrictions

In this section we will explore some alternatives to the relative weight restrictions of section 4, given that we, as stated previously, would like to avoid unreasonably high efficiency scores / cost norms due to the introduction of the geography variables. It is hard to see how one could specify meaningful bounds for the absolute levels of the geography weights, and we will therefore limit the discussion to virtual weight restrictions, which we believe is an interesting alternative. In section 5.1 we call attention to some challenges with the relative weight restrictions, and explain why virtual weight restrictions, as defined in section 3.3, represent an interesting alternative. In sections 5.2-5.4 we consider some alternatives with respect to virtual weight restrictions, and illustrate their effect on the DEA results. In section 5.5 we briefly discuss some other alternatives, before we conclude in section 5.6.

5.1 Why virtual weight restrictions?

Relative weight restrictions require choices to be made at a fairly detailed level in the model, as illustrated by the discussion in section 4. We need to specify which variables to include in the restrictions, as well as upper and/or lower bounds with respect to the ratio between their weights. Biases in the DEA results could arise if we make the wrong choices, e.g. by omitting relevant restrictions or by making false assumptions with respect to the bounds.

The non-completeness of the DEA model, due to the way the model was constructed, makes these challenges more severe. As described in NVE (2006a/b), statistical tests where used to check whether variables should be included in the model or not. Some cost drivers, such as low voltage lines, where excluded (mainly) because they did not pass the statistical tests, hence their effect on costs will be picked up by one or more of the cost drivers that *are* included, such as net stations or high voltage lines. However, it is difficult to know which of the included cost drivers are picking up the cost effect corresponding to an excluded driver. This fact makes it difficult to interpret the output weights in a meaningful way, and to relate the values of different output weights via restrictions. Specifically, it may be difficult to relate the geography weights to the weight of HV-lines, since we do not know what the normal level of the HV-weights should be, given that the HV-variable probably is picking up some of the effect of excluded cost drivers.

In order to reduce the need for detailed assumptions/choices, we propose alternative weight restrictions at a more aggregate level. Specifically, we group the outputs with respect to the type of output, and specify restrictions with respect to the percentage share of the cost norm that each group accounts for. The following groups are natural candidates for such restrictions:

1. Geography variables: forest, snow, coast
2. Product variables: delivered energy, customers, cottage customers

In the next three sections, we will analyze the effects of virtual weight restrictions with respect to these two variable groups, and compare with the effects of the relative weight restrictions in section 4. In order to limit the total weight of the geography variables we could add the following restriction to the LP-problem:

$$\frac{p_{Forest} \cdot y_{Forest,j^*} + p_{Snow} \cdot y_{Snow,j^*} + p_{Coast} \cdot y_{Coast,j^*}}{\sum_r p_r \cdot y_{rj^*}} \leq \alpha \qquad (5.1)$$

The number α has a value between 0 and 1, and represents the maximal share of the total cost norm (for the evaluated company j^*) that the geography variables may account for.

In order to use very high values for the geography weights, a company must use zero or very low values for some other outputs. In order to avoid this we could specify a lower bound for some of the remaining variables, e.g. on the product variables energy and customers. After all, the core activity of the distribution companies is to deliver energy and serve customers, and the role of the other output variables is to adjust for the fact that different companies perform these activities under very different conditions. Then it seems reasonable that at least some weight should be put on these “core” variables. We therefore specify the following lower bound for the product variables’ share of the cost norm:

$$\frac{p_{Energy} \cdot y_{Energy,j^*} + p_{Cust} \cdot y_{Cust,j^*} + p_{CCust} \cdot y_{CCust,j^*}}{\sum_r p_r \cdot y_{rj^*}} \geq \beta \qquad (5.2)$$

The number β has a value between 0 and 1, and represents the minimal share of the total cost norm that the product variables may account for.

5.2 Evaluation of alternative weight restrictions

In this subsection we will evaluate the effects of restrictions (5.1) and (5.2). We will start by looking at some particular cases with respect to the values of α and β for which we study the detailed effects of the restrictions. We would like to stress that we have no evidence to support the choice of particular values for α and/or β, and we will therefore provide some sensitivity analyses that may be of some help in making these choices. We also look at the combined effect of the two restrictions.

Maximum restriction - geography variables

We start by looking at the effect of (5.1) for the case α = 0.4, i.e., not more than 40 % of the cost norm can be accounted for by the geography variables. The effect with respect to the efficiency scores is illustrated in figure 5.1, where we have also included the unrestricted efficiency scores, as well as the efficiency scores resulting from implementing the relative restrictions described in section 4.2-4.3. As we saw from the sensitivity analysis in section 4.3, the relative restrictions affect a large number of companies. The virtual restriction, on the other hand, has a strong effect for a few companies, while most of the companies are (almost) unaffected.

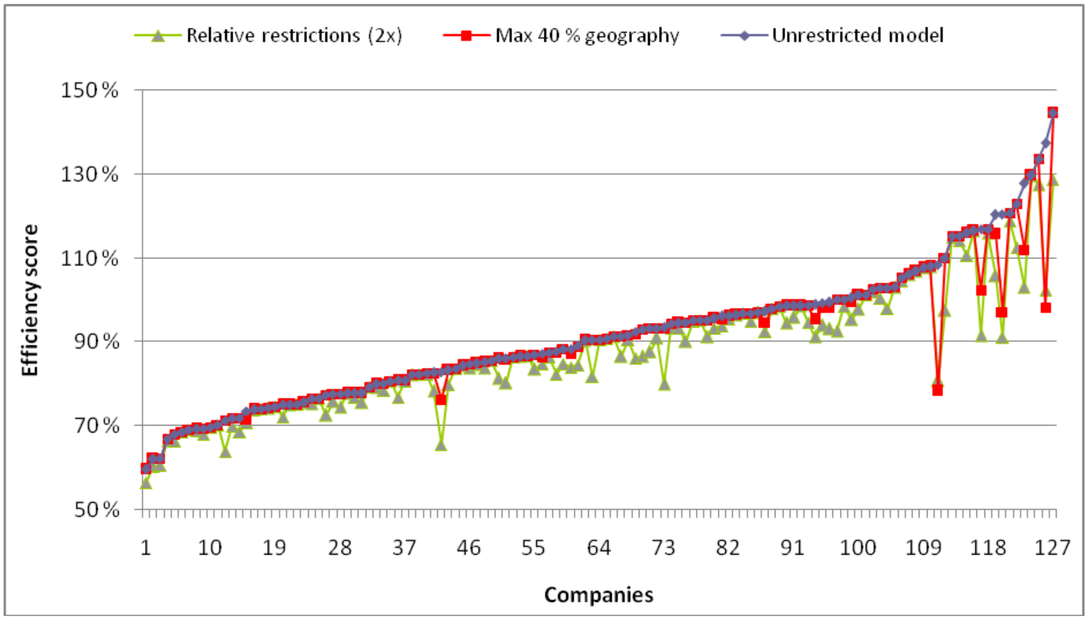

Figure 5.1: Effect of virtual weight restriction wrt geography (2006)

Figure 5.2 shows the 15 companies that are most affected by the virtual geography restricton. The unrestricted efficiency scores, as well as the efficiency score reductions caused by the weight restrictions, are shown in parentheses, and the cost norm shares of the outputs (virtual weights) are shown as the horizontal bars. We see that when the restriction is imposed, many companies choose to shift weight from the geography variables to either HV-lines or net stations. Only five companies experience reductions in their efficiency scores of 10 % or more.

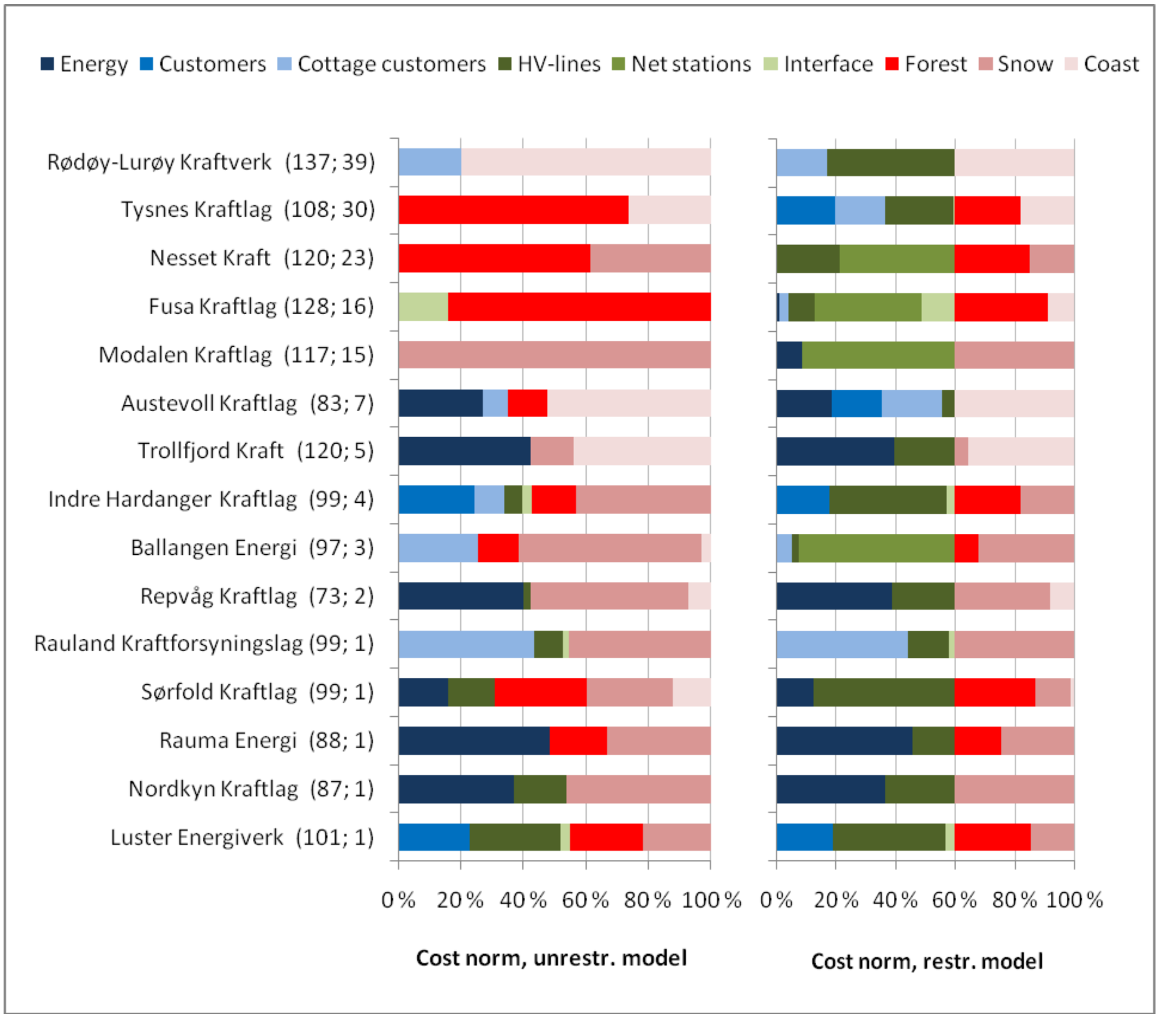

Figure 5.2: Companies most affected by the virtual weight restriction (2006)

In figure 5.3, we compare the relative weight restrictions from section 4.2 and 4.3 to the virtual weight restriction given by (5.1). The figure shows the 15 companies that are most affected by the relative weight restrictions. As we saw in section 4.3, the relative weight restrictions have a significant effect for a large number of companies. The virtual weight restriction, on the other hand, only have a significant effect for a few companies. The diagram in the middle illustrates that the relative restrictions are stronger than the virtual restriction, since the latter restriction is satisfied for all but two of the companies when the former restriction is imposed. This is not surprising, since the virtual restriction allows greater flexibility for the company with respect to how the weights should be adjusted in order to satisfy the restrictions. We see that in the case of the relative restrictions, the HV-variable make up for a large portion of the increased weight, while in the case of the virtual restriction the picture is more mixed.

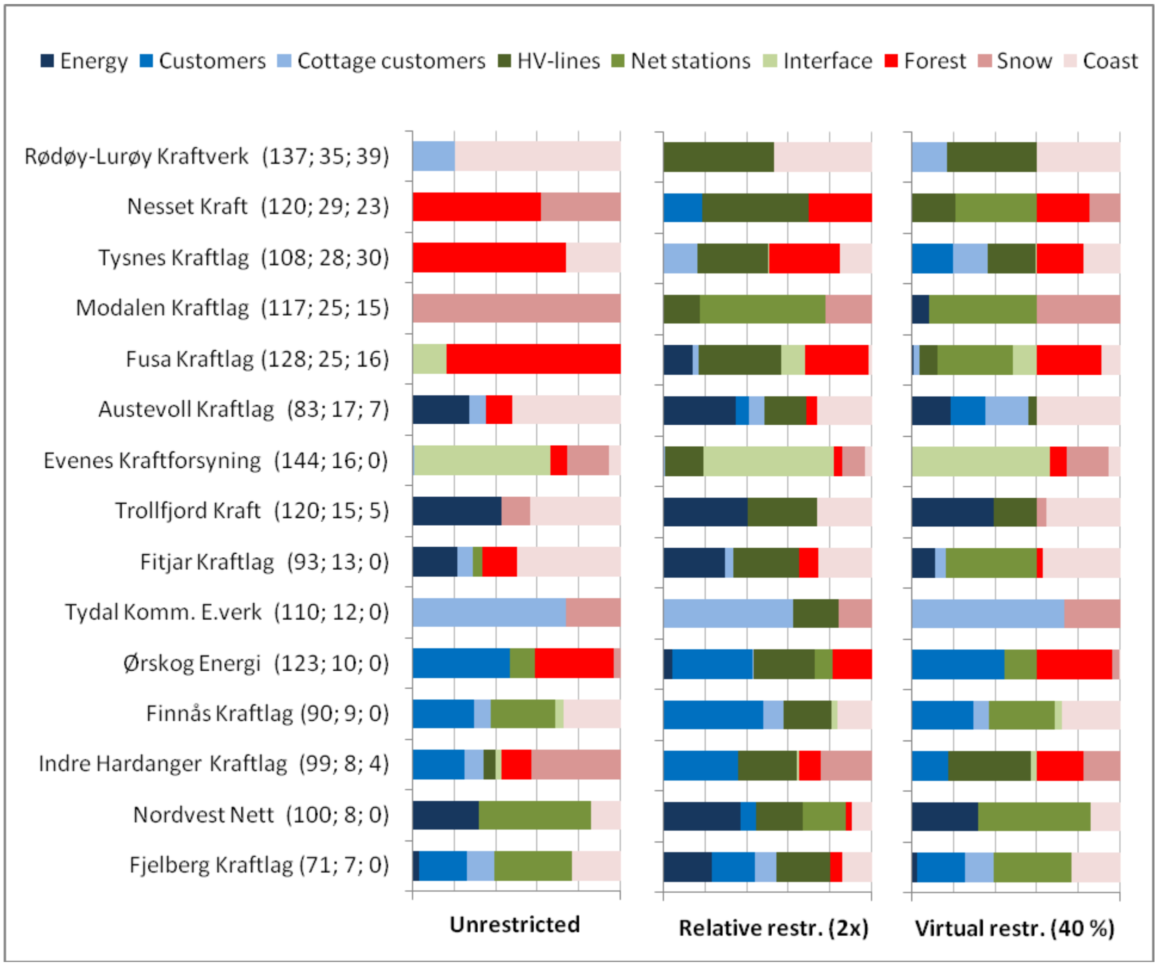

Figure 5.3: Comparison of relative and virtual weight restrictions (2006)

We conclude this section with a sensitivity analysis, shown in figure 5.4, with respect to the maximum share of geography in the cost norms. We see that, as in the case of the relative restrictions in section 4.3, the average cost-weighted efficiency is not much influenced by the weight restrictions. The number of affected companies drops dramatically as the maximum share is increased, but the average efficiency reduction per affected company does not change that much. Hence, it seems that virtual restrictions will to a greater extent punish companies that have chosen extreme weights, whereas the relative restrictions seem to affect a larger number of companies. Figure 5.4 also show the effect of the virtual weight restriction on the correlation between efficiency scores and two different measures of the geography variables' importance for individual companies. Again, as in section 4.3, we see that the positive correlation that is observed in the unrestricted model (weight limit equal to 100 %) is reduced as the restriction is tightened, and zero correlation is obtained for a limit between 20 % and 40 %, depending on how the correlation coefficient is defined. The correlation between efficiency scores and the virtual weights on the product variables energy / customers is negative in the unrestricted case, and becomes less negative as the restriction is tightened. As we saw in section 4.3, the correlation becomes zero only when the geography variables are removed from the model.

		Maximum geography share										
		0 %	**10 %**	**20 %**	**30 %**	**40 %**	**50 %**	**60 %**	**70 %**	**80 %**	**90 %**	**100 %**
Industry efficiency		89.8 %	91.8 %	92.8 %	93.2 %	93.4 %	93.5 %	93.6 %	93.6 %	93.6 %	93.6 %	93.6 %
No. of affected comp.		115	80	58	35	26	13	9	6	3	3	0
Average reduction		9.7	8.4	6.9	6.8	5.8	7.7	6.4	5.4	5.0	2.0	-
Reduction in %-points	**Over 50**	2	1	1	0	0	0	0	0	0	0	0
	25 - 50	8	6	4	3	2	1	0	0	0	0	0
	10 - 25	29	12	4	4	3	4	3	1	0	0	0
	5 - 10	22	19	13	1	1	0	2	2	2	0	0
	0 - 5	54	42	36	27	20	8	4	3	1	3	0
	No change	12	47	69	92	101	114	118	121	124	124	127
Corr(Eff, VirtualGeography)		-	-0.21	-0.08	0.05	0.12	0.17	0.22	0.25	0.29	0.30	0.32
Corr(Eff, PhysicalGeography)		-0.43	-0.30	-0.16	-0.05	0.02	0.06	0.10	0.13	0.14	0.15	0.16
Corr(Eff, VirtualProducts)		0.00	-0.01	-0.07	-0.10	-0.13	-0.14	-0.15	-0.15	-0.16	-0.17	-0.18

Figure 5.4: Sensitivity of efficiency scores w.r.t. max geography share (2006)

Minimum restriction - product variables

We will now look at the effect of (5.2), and we will start by looking at the case where $\beta = 0.3$, i.e., the product variables' share of the total cost norm cannot be smaller than 30 %. The effect of this restriction on the efficiency scores is shown in figure 5.5 below, and we see that there are a few companies with very large effects, and all of them are super-efficient prior to the introduction of the new restriction.

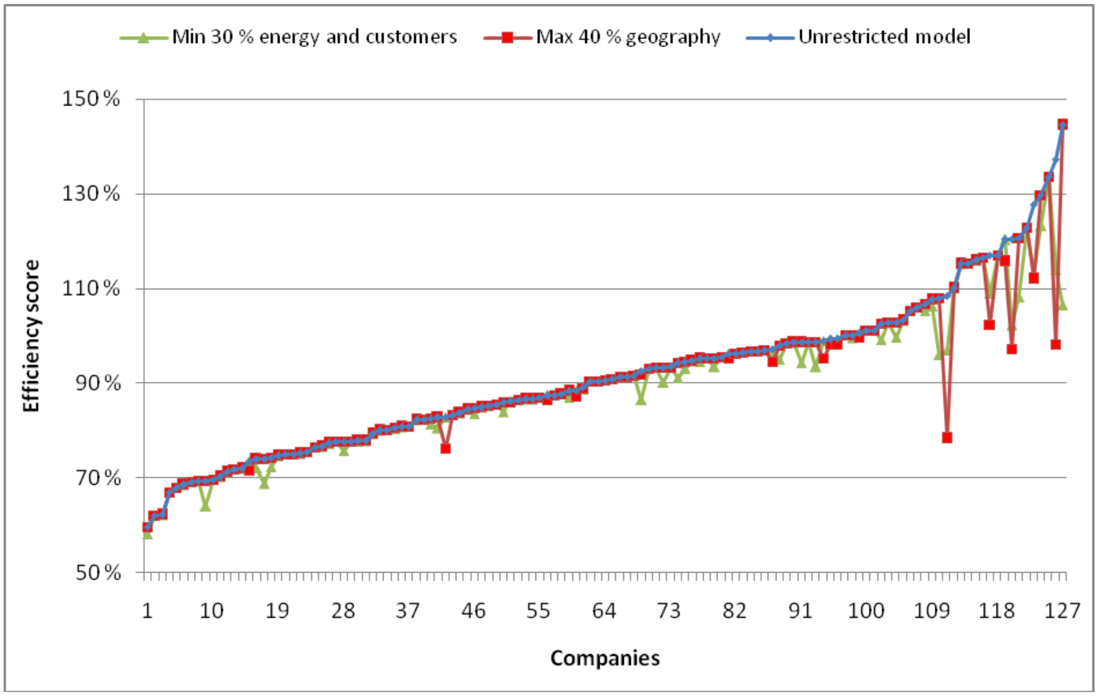

Figure 5.5: Effect of restriction (5.2) on efficiency scores (2006)

Figure 5.6 shows the identity of the companies that are most affected by the new restriction. Since less than 30 % of the norm was explained by energy and customers for all of these companies, the new restriction becomes binding, and they will all have cost norms, after the restriction is introduced, where exactly 30 % of the value comes from these outputs. The relative shares of the other outputs do not seem to change very much as a result of the new restriction.

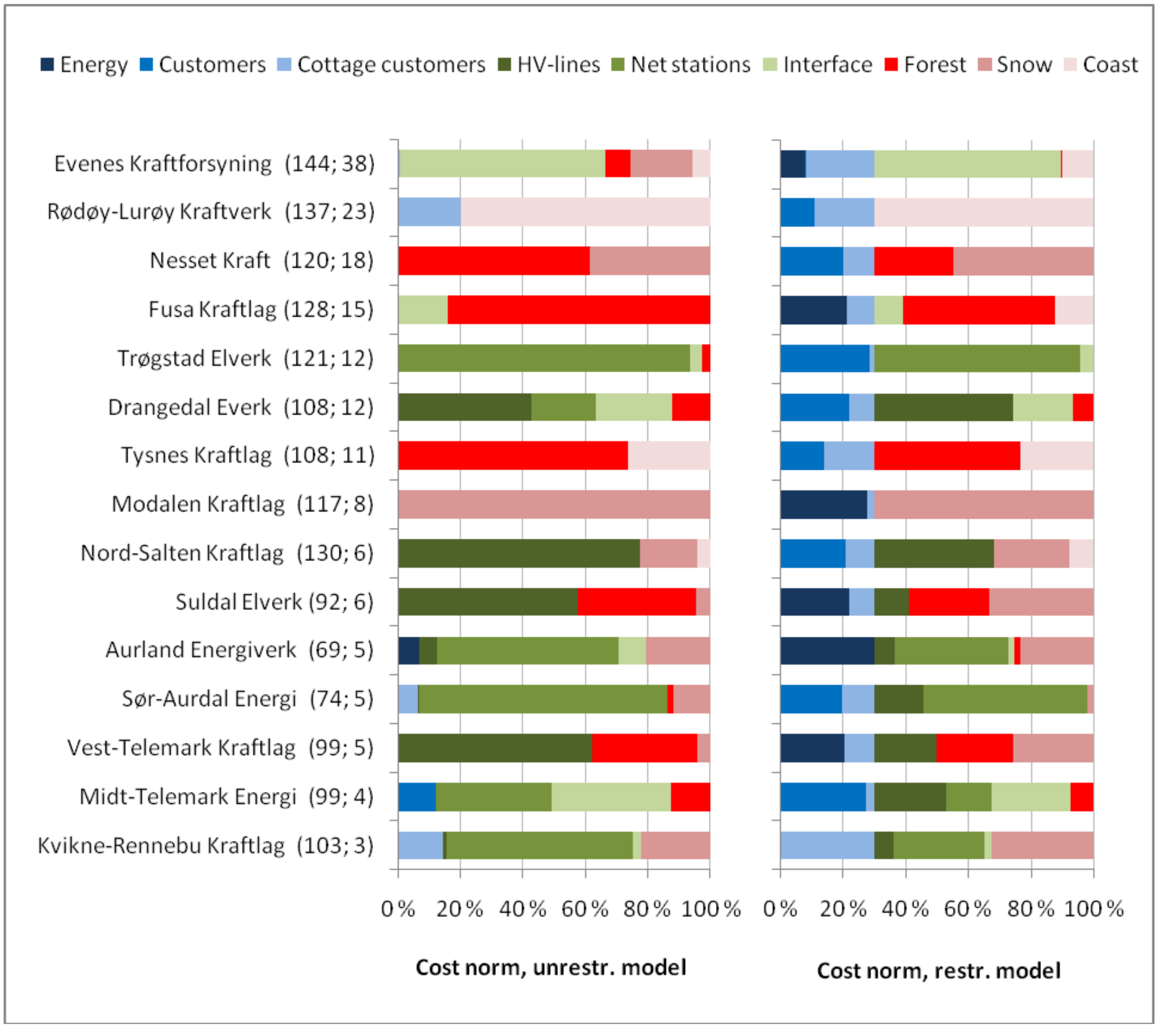

Figure 5.6: Companies most affected by restriction (5.2) (2006)

The sensitivity analysis in figure 5.7 below shows that the number of affected companies, as well as the average efficiency score reduction per affected company, increases considerably as the lower bound for the cost norm share is increased. This is not unexpected, since we have seen the same property in the case of the max-restriction for the geography variables in section 5.2. Again, as in the case of the other restrictions we have considered, the average cost-weighted industry efficiency is not much affected by changes in the restriction. Not surprisingly, the negative correlation between efficiency scores and the virtual weight on the product variables that we observe in the unrestricted model (limit equal to 0 %) is reduced as the restriction is tightened, and a similar effect is observed with respect to the correlation between efficiency scores and the geography variables.

		Minimum energy / customers share					
		0 %	**10 %**	**20 %**	**30 %**	**40 %**	**50 %**
Industry efficiency		93.6 %	93.6 %	93.5 %	93.2 %	92.8 %	91.7 %
No. of affected comp.		0	21	36	43	61	84
Average reduction		-	1.4	2.5	4.9	6.4	8.2
Reduction in %-points	**Over 50**	0	0	0	0	1	2
	25 - 50	0	0	1	1	2	6
	10 - 25	0	1	1	6	8	12
	5 - 10	0	0	3	6	8	20
	0 - 5	0	20	31	30	42	44
	No change	127	106	91	84	66	43
Corr(Eff, VirtualGeography)		0.32	0.30	0.26	0.19	0.12	-0.02
Corr(Eff, PhysicalGeography)		0.16	0.15	0.13	0.09	0.03	-0.07
Corr(Eff, VirtualProducts)		-0.18	-0.14	-0.09	-0.01	0.09	0.17

Figure 5.7: Sensitivity analysis w.r.t. virtual restrictions on product variables (2006)

Combination of max and min virtual restrictions

The restrictions given by (5.1) and (5.2) may also be combined in the same model. We illustrate the combined effect, for α = 0.4 and β = 0.3, in figure 5.8 and 5.9 where we compare the combined effect to the effect of using only (5.1). The identity of the most affected companies, as well as their efficiency score reductions (in parentheses), are shown in figure 5.9. We see that the combined restrictions have a stronger effect than the max-restriction alone, and for one company (Evenes) the difference is dramatic.

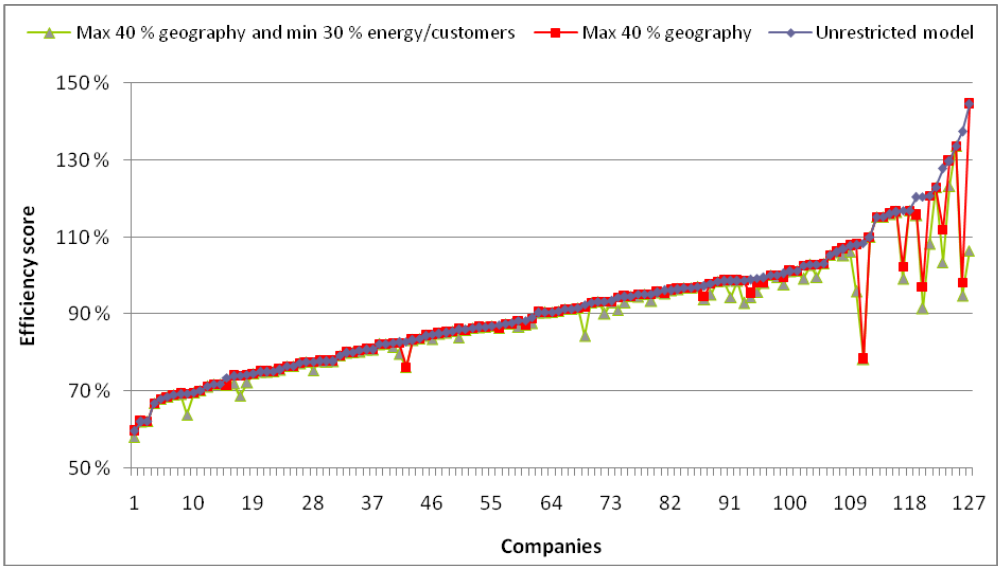

Figure 5.8: Effect of combined restrictions (2006)

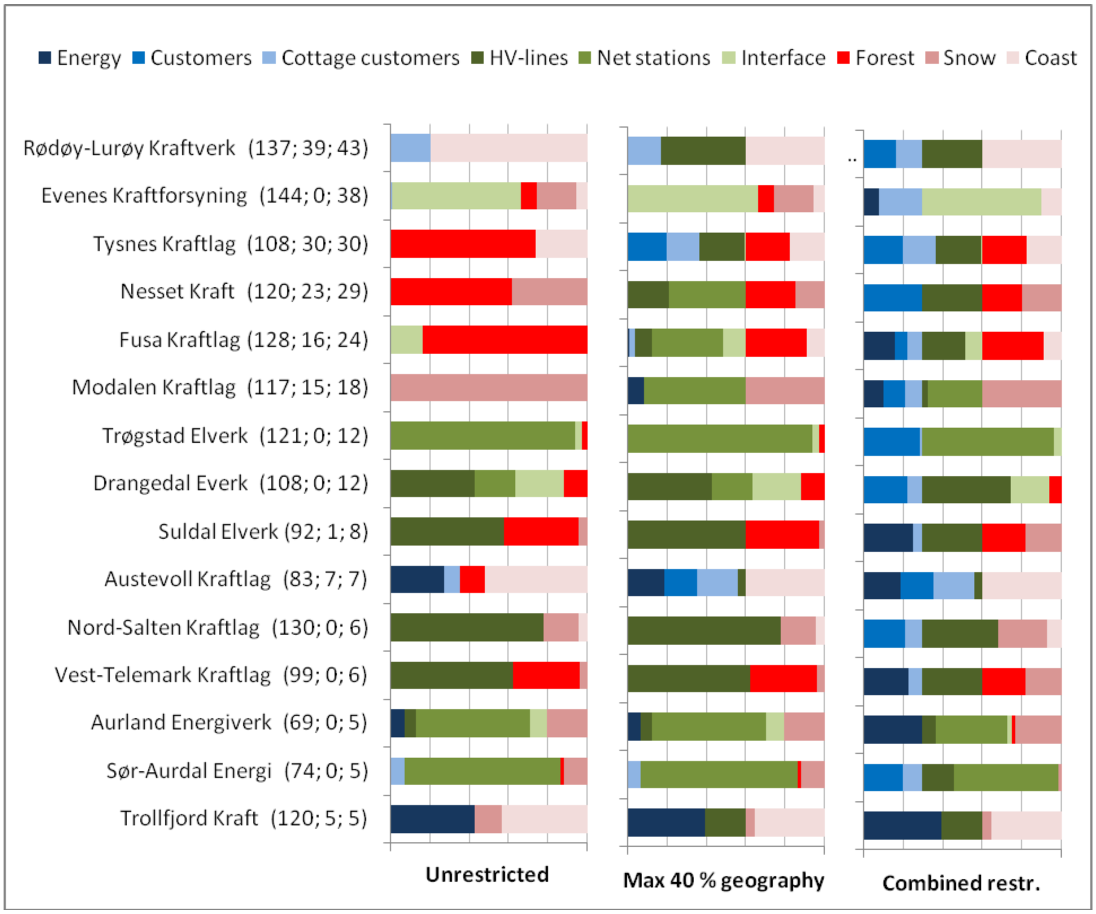

Figure 5.9: Companies most affected by combined restrictions (2006)

5.3 Weight restrictions and reference companies

Figure 5.10 below lists all the companies that appear in the reference sets in the 2006 data set and illustrates the relative contribution of each reference company to the total cost norm of the industry. The cost norm contribution of company *j* is computed as its cost (x_j) times the sum of its weight (λ_j) in all the reference sets of which it is a member. The columns of the diagram correspond to different versions of the DEA model. The leftmost column corresponds to the unrestricted model, and columns 2-4 correspond to some of the weight restrictions that we have discussed previously. Next to the company names we have indicated their virtual weights on geography and energy/customers, respectively, in the unrestricted case. We see that a relatively small number of companies explain a large share of the industry cost norm. In the unrestricted case, for instance, more than 80 % of the cost norm is explained by 10 companies. Out of these 10 companies, one has a virtual geography weight of more than 40 %, and three have a virtual weight on energy/customers lower than 30 %. We see that the introduction of weight restrictions does not have dramatic effects on the composition of the cost norm, although there are some differences.

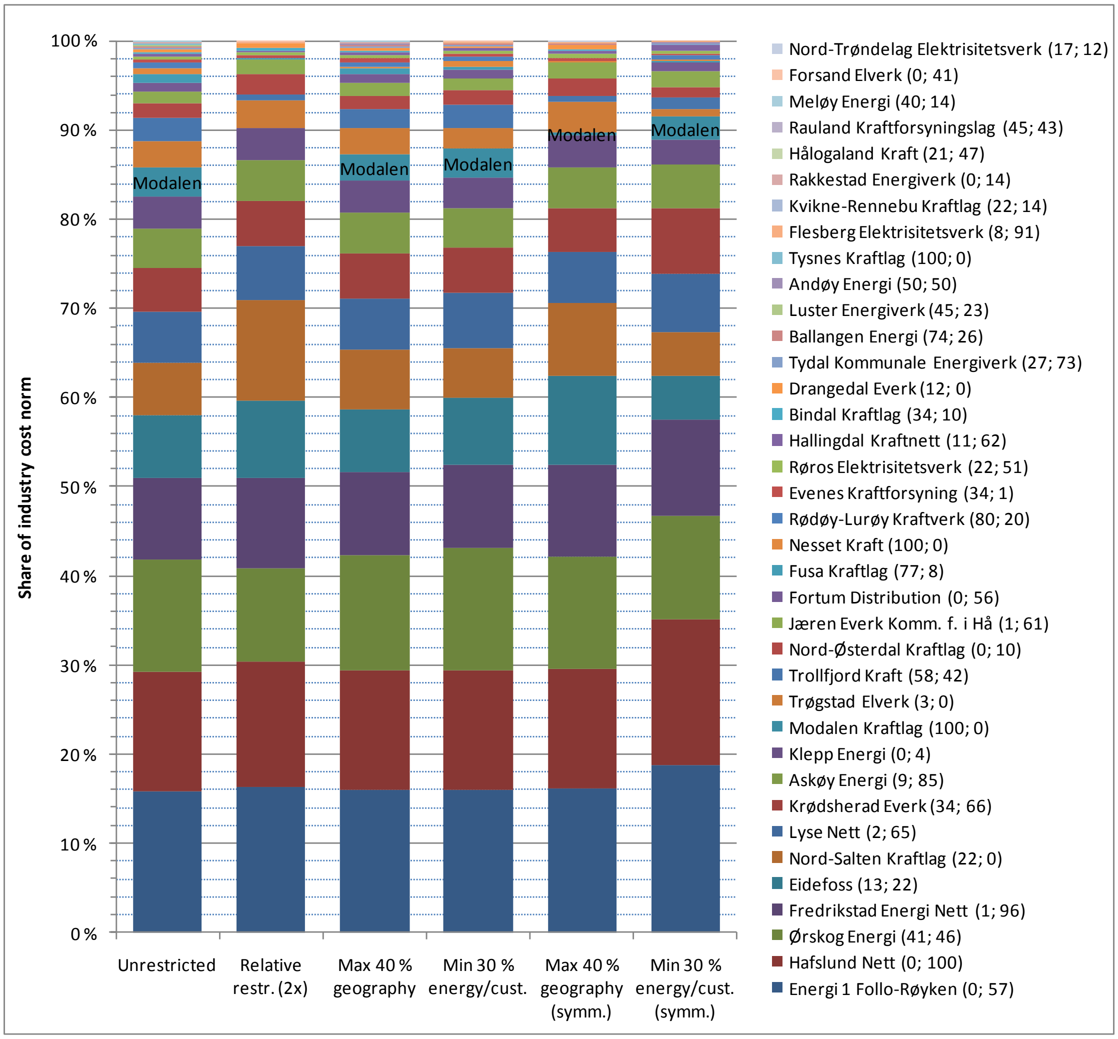

Figure 5.10: Reference companies in the 2006 data set

Figures 5.11 and 5.12 show the ratio between each reference company's contribution to the industry cost norm, and its own actual cost, where the companies have been sorted according to the virtual geography weight and the virtual weight on energy/customers, respectively. The highest observed ratio is for Modalen, whose cost norm contribution is 148 times as high as its own actual cost. Figures 5.11 and 5.12 show the names of all companies with a ratio of 10 or more, and we see that all these 9 companies appear among the 12 most important reference companies in figure 5.10. Together they account for over 60 % of the industry norm[15]. From the figures we see some indication that companies with high (low) weight on geography (energy/customers) explain more of the cost norm, relative to their own actual cost, than other companies, but the picture is somewhat mixed.

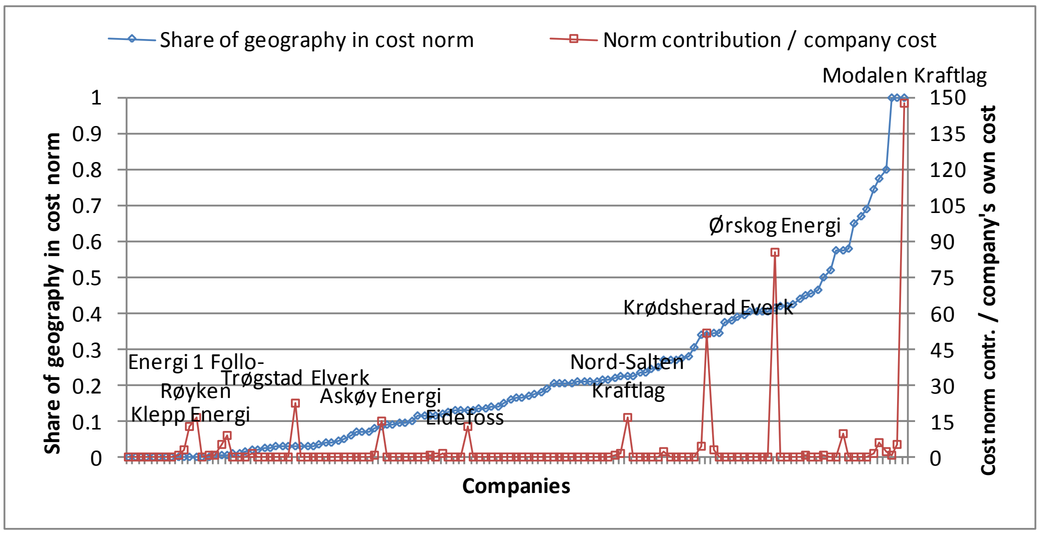

Figure 5.11: Reference companies and virtual weights on geography (2006)

[15] The relatively large weight of small companies in the industry cost norm is a problem in itself, since it makes the DEA results vulnerable to the numbers reported by those companies, and this problem was discussed by Bjørndal and Bjørndal (2006a).

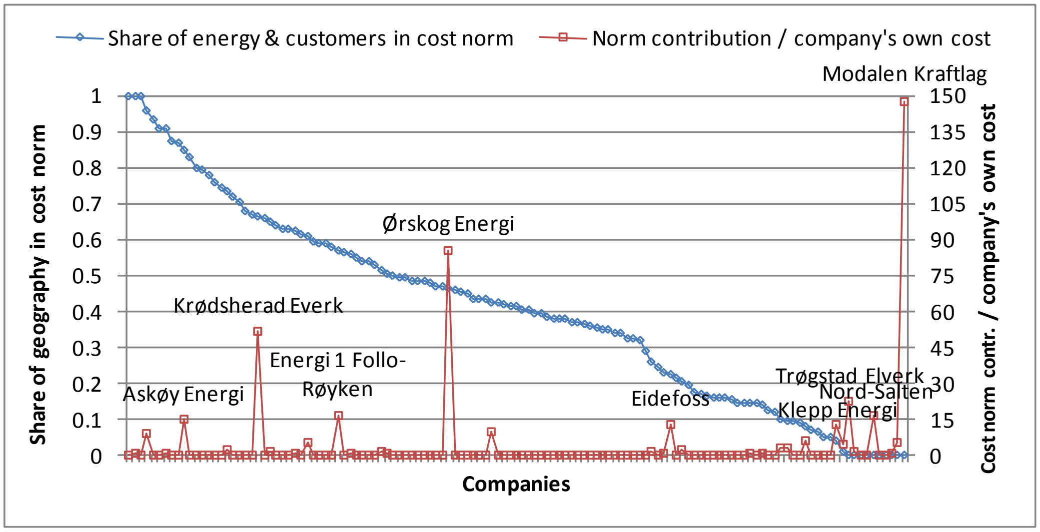

Figure 5.12: Reference companies and virtual weights on energy/customers (2006)

We will use the example of Modalen to illustrate an important difference between the relative and virtual weight restrictions. Modalen has all of its weight on geography, and we saw in the previous section that its efficiency score was significantly reduced (-15 and -8 %-points) when we introduced the virtual weight restrictions given by (5.1) and (5.2), with α = 0.4 and β = 0.3, respectively. However, figure 5.10 shows that Modalen's role as reference company is not visibly affected by the virtual weight restrictions. The relative weight restrictions, on the other hand, will reduce Modalen's efficiency score dramatically (-25 %-points) *and* eliminate the company from the reference sets. The reason for this phenomenon is that the virtual weight restrictions given by (5.1) and (5.2) apply only to the company that is being evaluated, and not to the potential reference companies. Figure 5.13 below illustrates this. The numbers in the table are based on a DEA model with a weight restriction according to (5.1) and with $\alpha = 0.4$. We have chosen Modalen and Sunndal as examples, since Modalen is one of the reference companies of Sunndal. The table illustrates that when the cost norm of a company is evaluated based on the company's own DEA weights, the virtual weight restriction is satisfied. For Modalen, the virtual weight on the geography variables is 40 %, i.e., exactly at the upper bound, whereas for Sunndal the corresponding weight is 38.8 %. However, if we evaluate Modalen with the weights of Sunndal, for which Modalen serves as reference company, the restriction is not satisfied, since 66.6 % of Modalen's cost norm then is explained by the geography factors. Hence, when Modalen appears in the reference set of

Sunndal, it is not subject to the same weight restriction as when its own efficiency is evaluated. Virtual weight restrictions in the form of (5.1) are company-specific, since the output weights are multiplied by the physical quantities (the y's) of the company that is being evaluated. The relative weight restrictions, on the other hand, are not company-specific, since they only involve the output weights (the p's).

	Modalen norm based on Modalen weights		Sunndal norm based on Modalen weights		Sunndal norm based on Sunndal weights		Modalen norm based on Sunndal weights	
Output	NOK	Relative	NOK	Relative	NOK	Relative	NOK	Relative
Energy	184 476	8.7 %	2 461 515	16.3 %				
Customers					11 000 461	61.2 %	694 876	33.4 %
Cottage customers								
HV-lines								
Net stations	1 091 987	51.3 %	9 281 886	61.4 %				
Interface								
Forest					1 549 532	8.6 %	17 913	0.9 %
Snow	850 975	40.0 %	3 377 971	22.3 %	5 421 689	30.2 %	1 365 826	65.7 %
Coast								
Sum	2 127 437	100.0 %	15 121 372	100.0 %	17 971 682	100.0 %	2 078 615	100.0 %
Virtual geography weight		*40.0 %*		*22.3 %*		*38.8 %*		*66.6 %*

Figure 5.13: Alternative cost norms for Modalen (2006)

The apparent asymmetry of the virtual weight restrictions should not necessarily be seen as a problem. The purpose of these restrictions is to avoid very high efficiency scores as a result of unreasonable weighting of the output variables. Such unreasonable weights occur when the evaluated company has a special output profile, making it difficult to find comparable companies that can serve as reference. In a model where super efficiency is allowed, i.e., a company is not allowed to be its own reference, this will be an even bigger problem. It seems plausible to restrict the company's *own* weights based on this argument, as we have done in (5.1) and (5.2), but it is not obvious that it should lead to restrictions on the weights of the *other* companies[16]. In other words, the fact that a company is special should not be rewarded with an unreasonably high efficiency score, but it should not prevent the company from being a reference for other companies!

[16] Beasley and Wong (1990) argue that such restrictions could be more in line with the logic behind the basic DEA model, i.e., that each company is free to choose its weights as it wants, but subject to constraints with respect to the effect of these weights on the efficiciency scores of other companies.

If asymmetry really is seen as a problem, we could replace (5.1) by the following set of restrictions[17]:

$$\frac{p_{Forest} \cdot y_{Forest,j} + p_{Snow} \cdot y_{Snow,j} + p_{Coast} \cdot y_{Coast,j}}{\sum_r p_r \cdot y_{rj}} \leq \alpha \quad \text{for all } j=1,\dots,n. \quad (5.3)$$

Since the restriction in (5.1) is included in (5.3), the latter set of restrictions will be stronger than the former[18]. This is illustrated by figure 5.14 below, where we see that the symmetric version yields efficiency scores less than or equal to the efficiency scores with the company-specific restriction. Figure 5.15 illustrates this for different values of the restriction limit. We see that the symmetric version will affect a larger number of companies. The maximum effect is not very different for the two versions, but the average effect is smaller for the symmetric version in all but one case. With respect to the correlation between efficiency scores and the physical geography measure, the symmetric restriction has a stronger effect than the company-specific restriction.

[17] Another alternative, suggested by Wong & Beasley (1990), is to replace the output quantities in (5.1) by *average* quantities. See also Sarrico & Dyson (2004) or Thanassoulis et al. (1997) for a discussion of the various alternatives.

[18] In general, stronger restrictions can cause the LP-problem to become infeasible. This will not occur, however, if the problem only includes restrictions of the type given by (5.3), since the restrictions can always be satisfied by setting some output weights equal to zero. This would be equivalent to removing the corresponding output variables from the DEA model.

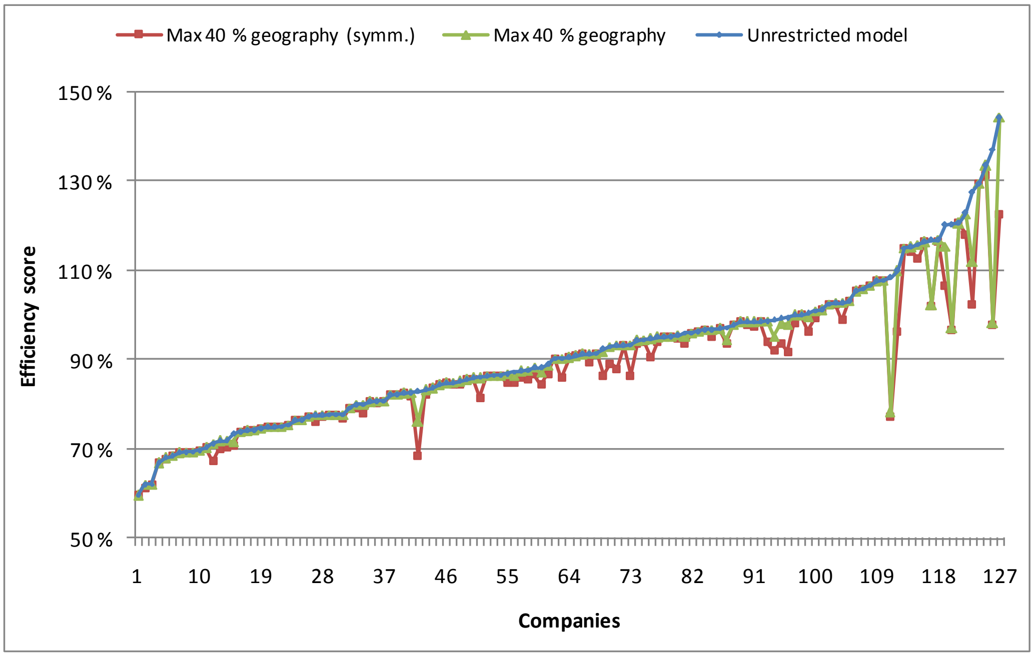

Figure 5.14: Symmetric versus company-specific virtual weight restrictions (2006)

	Maximum geography share										
	0 %	**10 %**	**20 %**	**30 %**	**40 %**	**50 %**	**60 %**	**70 %**	**80 %**	**90 %**	**100 %**
No. of affected comp.											
Company-specific	115	80	58	35	26	13	9	6	3	3	0
Symmetric	115	112	99	95	80	57	37	23	12	6	0
Average reduction											
Company-specific	9.7	8.4	6.9	6.8	5.8	7.7	6.4	5.4	5.0	2.0	-
Symmetric	9.7	8.1	7.1	5.4	4.1	3.5	3.0	2.7	2.7	2.0	-
Maximum reduction											
Company-specific	69.4	63.7	57.0	49.0	39.1	26.8	17.1	10.4	6.2	3.0	0.0
Symmetric	69.4	64.2	57.5	49.3	39.3	27.2	17.1	12.4	8.6	4.0	0.0
Corr(Eff, PhysicalGeography)											
Company-specific	-0.43	-0.30	-0.16	-0.05	0.02	0.06	0.10	0.13	0.14	0.15	0.16
Symmetric	-0.43	-0.35	-0.26	-0.17	-0.07	0.02	0.08	0.11	0.14	0.15	0.16

Figure 5.15: Sensitivity analyses for company-specific and symmetric weight restrictions

5.4 Which companies are punished – one or several geography factors?

Some companies may differ with respect to the number of geography factors that they are exposed to. A possible objection that has been put forward against virtual weight restrictions á la (5.1) is that they may punish companies that are exposed to several geography factors in an unfair manner, since the restriction is formulated with respect to the sum of cost norm shares for the geography variables. In order to investigate whether this is true, we have looked at the values of the geography variables for the companies most affected by restriction (5.1) with $\alpha = 0.4$. Figure 5.16 shows companies with efficiency score reductions of at least 0.5 percentage points. We also show the ratio between the maximum and average value of the three geography variables for each company[19]. If it is true that companies with large efficiency score reductions tend to have relatively high values of *several* geography variables, then the maximum value should be relatively low compared to the average for these companies. However, the diagram does not indicate that this is true. Figure 5.17 shows statistics for the geography variables for affected and non-affected companies, with respect to both virtual and relative weight restrictions. The numbers show, not surprisingly, that affected companies have higher values for the geography variables than non-affected companies. However the ratio between maximum and average values of the geography variables does not seem to be lower for affected than for non-affected companies. For the 2006 data set, the opposite is in fact true, both for the virtual and relative weight restrictions. A possible explanation for this result is that an unrestricted DEA model in itself tends to favor companies that have “extreme” output combinations, i.e., that have very high values for one output factor rather than moderate amounts of several factors. Hence, companies in the former category tend to get punished harder by *any* type of weight restriction than companies belonging to the latter category.

[19] I.e., for each company we have computed the ratio Max(Forest, Snow, Coast) / Average(Forest, Snow, Coast).

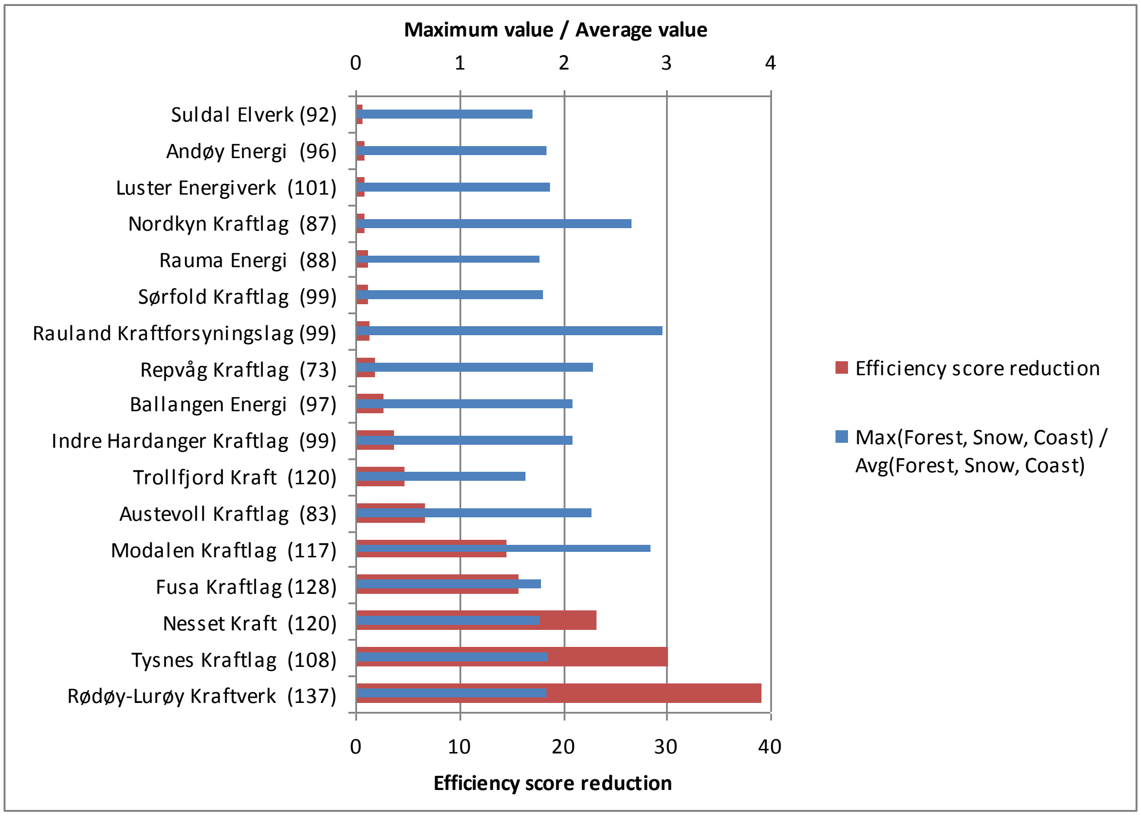

Figure 5.16: Companies most affected by virtual weight restriction (2006, $\alpha = 0.4$)

	No. of companies	Forest / HV	Snow / HV	Coast / HV	Max (Forest, Snow, Coast) / Avg(Forest, Snow, Coast)
Affected	26	0.157	0.332	0.133	1.993
Non-affected	101	0.093	0.193	0.043	1.960

(a) Virtual restriction (max 40 % geography)

	No. of companies	Forest / HV	Snow / HV	Coast / HV	Max (Forest, Snow, Coast) / Avg(Forest, Snow, Coast)
Affected	91	0.112	0.254	0.078	2.018
Non-affected	36	0.093	0.140	0.020	1.835

(b) Relative restriction (2x)

Figure 5.17: Affected and non-affected companies (2006, $\alpha = 0.4$)

5.5 Summary and conclusions

Since the DEA model is not complete, meaning that it does not contain all the relevant cost drivers, it will be difficult to interpret the weights and to relate them via relative weight restrictions. We have therefore formulated two types of virtual weight restrictions defined on *groups* of variables: an upper bound for the percentage share of geography variables in the cost norm of any company, as well as a lower bound for the share of the product variables. Because they are defined with respect to groups of variables, they allow for more flexibility in setting the weights than the relative restrictions, and our analyses indicate that the level of the restrictions can be set such that the effects for most companies are relatively minor, while companies with extreme weighting schemes are punished. Such a property should fit well in with the intentions behind NVE's proposal, namely to avoid unreasonably high efficiency scores as a result of the geography variables. An important feature of the virtual weight restrictions given by (5.1) and (5.2) at is that they are company-specific, and it may be argued that a symmetric version should be used. We question whether the asymmetry really poses a problem, but we also discuss the consequences of imposing symmetric restrictions. We also test the hypothesis that virtual restrictions with respect to groups of output variables will punish companies with combinations of several geography factors harder than companies with only one geography factor, but we find no support for it.

6. Determination of limits

As the sensitivity analyses in section 5 clearly demonstrate, the effect of weight restrictions depends on the exact limits that are determined. In the literature (see for instance Thanassoulis et al. (2004)) various methods are mentioned, including using

- Shadow prices from unrestricted DEA models
- Shadow prices for a subset of model companies
- Information about prices or costs
- Average values from e.g. regression analysis
- Expert opinions

6.1 Shadow prices and degeneracy

One possibility is to use information on the shadow prices in the unrestricted DEA models. This however, is a method that should be used with caution. One reason for that is degeneracy in the LP solutions where efficiency scores are computed. If the LP problem is degenerate, there will be multiple dual solutions, i.e. multiple sets of shadow prices that all solve the dual LP problem. In the data set from 2006, there is some degeneracy connected to the interface variable, and figure 6.1 illustrates by showing the lowest/highest possible values of the optimal interface weight for each company. The average difference is NOK 410, while the maximal difference is NOK 4 907. Figure 6.2 shows a summary of similar computations for 2005 and 2006, and we see that the interface weight is the only non-unique weight in both years. The non-uniqueness of the interface weight occurs for companies which have nothing of the interface variable, and for which all the reference companies have nothing of it as well[20]. There were 12 non-unique instances in 2005 and 38 in 2006.

[20] Note that the *virtual* interface weights are unique for all companies, since the virtual output quantities are found by multiplying physical quantities by the corresponding output weights.

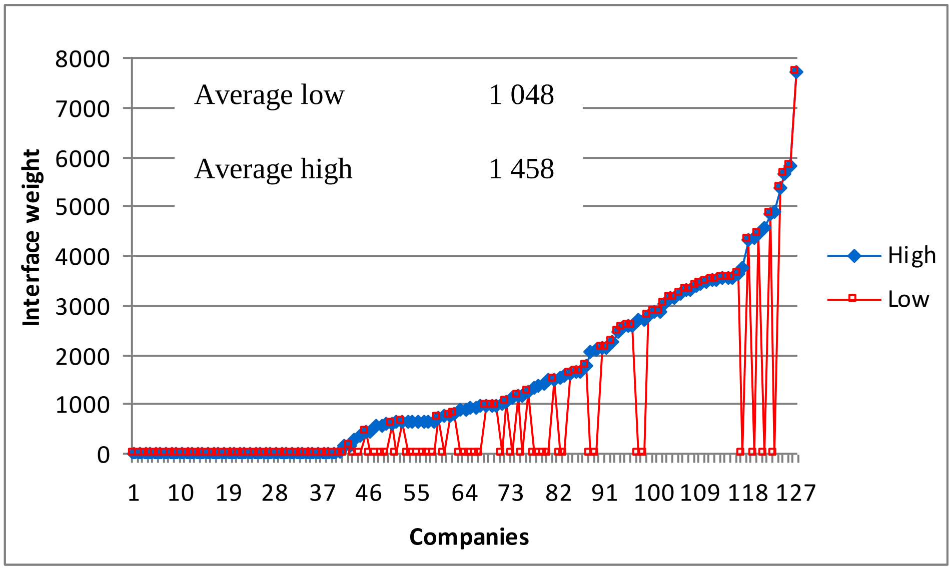

Figure 6.1: Optimal weights for the interface variable (2006)

	2005				2006			
	Low average	**High average**	**Average diff.**	**Max diff.**	**Low average**	**High average**	**Average diff.**	**Max diff.**
Energy	21	21	0	0	32	32	0	0
Customers	605	605	0	0	510	510	0	0
Cottage customers	1 531	1 531	0	0	1 165	1 165	0	0
HV-lines	4 864	4 864	0	0	8 735	8 735	0	0
Net stations	15 979	15 979	0	0	12 896	12 896	0	0
Interface	996	1 197	201	4 956	1 048	1 458	410	4 907
Forest	29 284	29 284	0	0	28 184	28 184	0	0
Snow	18 445	18 445	0	0	24 193	24 193	0	0
Coast	22 847	22 847	0	0	22 700	22 700	0	0

Figure 6.2: Optimal low/high weights for all variables

6.2 Information about prices or cost

Another alternative is to use information about prices or costs, for instance from regression analyses or expert opinions. Geography factors will influence the annual cost of capital, through investment expenditures and economic life. Moreover, geography factors will affect annual operating and maintenance costs. The challenge is to find appropriate multipliers to take into consideration the difference between “easy” and "difficult" geographic locations, along the dimensions that are not taken into account by other output variables (like customer density).

How specifically to determine the limits, and which level of detail that is necessary, must also depend on what kind of limit we consider, whether it is absolute, relative or virtual weight restrictions.

Relative weight restrictions

If we for instance are considering relative weight restrictions, it can be useful to start by investigating what effect changes in outputs (or partial outputs) will have on the cost norms, and how the shadow prices can be interpreted. For this purpose, we will use the (unrestricted) weights for Nord-Trøndelag Elektrisitetsverk AS in 2006, shown in figure 6.3. This company has positive weights for energy, net stations, as well as the forest and snow variables. Net stations account for most of the cost norm, with a virtual weight of 70.5 %. The last two columns of the table show how much the output quantities may be altered without causing changes in the output weights.

	Physical quantity	**Slack**	**Weight (NOK)**	**Cost norm (1000 NOK)**	**Share of cost norm**	**Allovable increase**	**Allowable decrease**
Energy	1 934 568.0		22	42 739	12.1 %	1 993 620.0	1 756 380.0
Customers	70 517.0	3 177.5				73 697.5	INF
Cottage customers	8 487.0	10 777.1				19 263.9	INF
HV-lines	5 123.0	347.8				5 470.7	INF
Net stations	6 511.0		38 284	249 267	70.5 %	8 599.1	6 322.4
Interface	0.0	643.5				643.4	INF
Forest	670.2		16 206	10 861	3.1 %	868.7	497.1
Snow	1 296.9		39 081	50 685	14.3 %	1 747.2	1 180.9
Coast	43.3	82.9				126.2	INF
Sum				353 551	100.0 %		

Figure 6.3: Unrestricted DEA results for Nord-Trøndelag Elektrisitetsverk (2006)

Figure 6.4 illustrates the effect of installing 1 kilometer of additional high voltage lines (through air). The effect on the cost norm will depend on the type of line that is installed. HV-lines have zero weight, so the only way that additional lines can influence the cost norm is via the geography variables, since the geography indices are scaled with HV-lines. Note that if the new line is a sea/ground cable, the effect on the cost norm will be zero, since only air cables are included in the scaling factor. Since both the forest variable and the snow variable have positive weight, the effect on the cost norm will indeed be positive in this case. One kilometer of additional HV-lines causes the forest and snow variables to increase by 0.154 and 0.298, respectively, which correspond to the values of the geography indices, where we have normalized the values as described in section 4.2. Multiplying the increments by the corresponding output weights of NOK 16 206 and NOK 39 081, respectively, gives us a total increase in the cost norm of NOK 14 132. Note that these calculations are only valid if the total increase, measured relative to the allowable increase of the output quantities, is less than 100 %-points[21].

	New physical quantity	**Increase**	**% of allowable increase**	**Weight (NOK)**	**Increase in cost norm (NOK)**
Energy	1 934 568.0			22	
Customers	70 517.0				
Cottage customers	8 487.0				
HV-lines	5 124.0	1.000	0.02 %	0	0
Net stations	6 511.0			38 284	
Interface	0.0				
Forest	670.3	0.154	0.02 %	16 206	2 494
Snow	1 297.2	0.298	0.02 %	39 081	11 638
Coast	43.3	0.010	0.01 %	0	0
Sum			0.06 %		14 132

Figure 6.4: Effect on the cost norm of installing 1 extra kilometer of HV-line, without relative weight restrictions (NTE, 2006)

We repeat the analysis in the above example for the case where the DEA model includes weight restrictions. Note that the geography variables have a total weight of 17.4 % for this company. Hence, the marginal output values for this company would not be affected by a moderate restriction on the virtual weights. In Figure 6.5 we show the weights that result when we restrict the weights of each of the geography variables to be less than or equal to 2 times the weight of HV-lines. Figure 6.6 illustrates the effect of installing an extra kilometer of HV-lines, given that the new line is through air, and hence will influence the value of the geography factors. The weight of HV-lines is now positive and

[21] See chapter 5 in Hillier and Hillier (2008) for an explanation of "The 100 Percent Rule".

equal to NOK 4 228, and the extra kilometer of line will cause the cost norm to increase by this amount. The values of the increased forest/snow variables have to be added to this amount, and the total increase in the cost norm will be NOK 8 047. Note that if we had installed a sea/ground cable instead of air cable, the geography factor would not have been affected, and the total marginal value of the extra line length would be only NOK 4 228. Figure 6.7 summarizes the effect on the cost norms of installing an extra kilometer of line under various assumptions with respect to the type of line installed and the type of DEA model that is used. It is interesting to note that, for this particular company, relative investment incentives are clearly influenced by the introduction of the relative weight restrictions.

	Physical quantity	**Slack**	**Weight (NOK)**	**Cost norm (1000 NOK)**	**Share of cost norm**	**Allowable increase**	**Allowable decrease**
Energy	1 934 568.0		51	98 027	28.2 %	2 022 730.0	1 897 640.0
Customers	70 517.0		177	12 508	3.6 %	73 850.3	67 949.5
Cottage customers	8 487.0	10 777.9		0		19 264.9	INF
HV-lines	5 123.0		4 228	21 659	6.2 %	5 320.3	4 607.1
Net stations	6 511.0		30 612	199 313	57.3 %	6 824.7	6 316.2
Interface	0.0	680.9		0		680.9	INF
Forest	670.2		8 456	5 667	1.6 %	768.8	412.2
Snow	1 296.9		8 456	10 966	3.1 %	1 395.6	1 128.6
Coast	43.3	32.6				75.9	INF
Sum				348 139	100.0 %		

Figure 6.5: Results from a DEA model with relative weight restrictions (NTE, 2006)

	New physical quantity	Increase	% of allowable increase	Weight (NOK)	Increase in cost norm (NOK)
Energy	1 934 568.0			51	
Customers	70 517.0			177	
Cottage customers	8 487.0				
HV-lines	5 124.0	1.000	0.02 %	4 228	4 228
Net stations	6 511.0			30 612	
Interface	0.0				
Forest	670.3	0.154	0.02 %	8 456	1 301
Snow	1 297.2	0.298	0.02 %	8 456	2 518
Coast	43.3	0.010	0.01 %	0	0
Sum			0.07 %		8 047

Figure 6.6: Installing an extra kilometer of line (through air), restricted DEA model (NTE, 2006)

		DEA model	
		Unrestricted	Restricted (2x)
Type of line	Ground/sea cable	0	4 228
	Air cable	14 132	8 047

Figure 6.7: Summary of incremental effects in various cases (NTE, 2006)

Virtual weight restrictions

With virtual weight restrictions on the geography variables, we are concerned with how large share of the total cost norm that can be determined by the geography variables. As shown in the examples in section 5, this may be complemented with lower bounds on the virtual weights on product variables like delivered energy and the number of customers. A starting point for determining the specific limits could be the average cost allocated to different cost groups, as illustrated in figure 6.8.

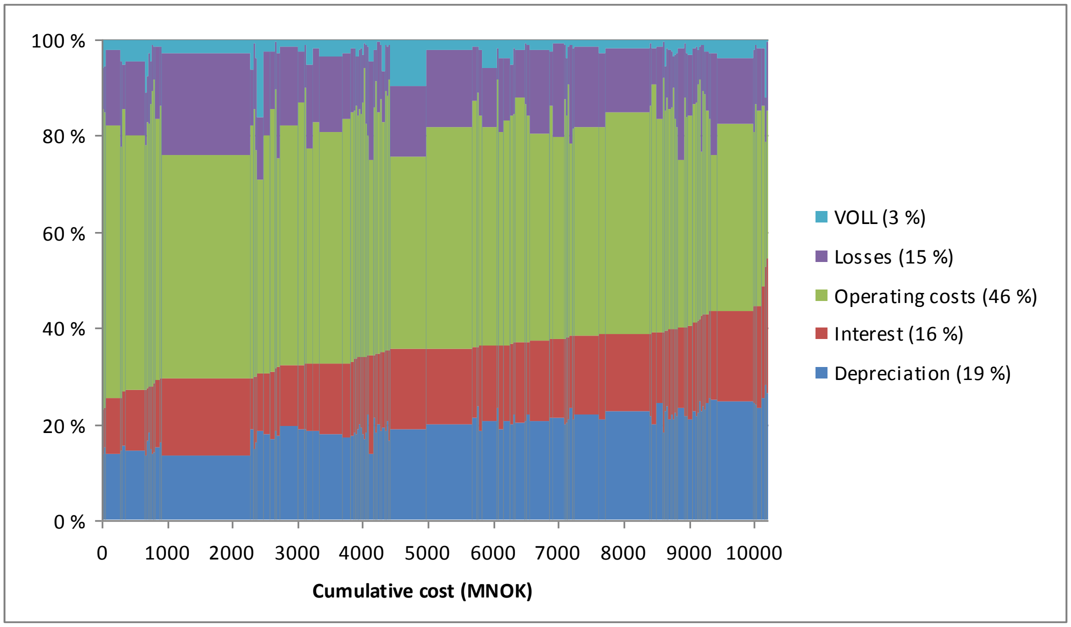

Figure 6.8: Cost allocated to cost groups (2006)

Combined with an evaluation of the maximal effect of geography variables on annual cost within the different cost groups, this may serve to establish limits on total cost shares. It would also be useful to establish cost groups where geography factors do not influence cost. Such a cost group could be customer related cost, i.e. cost for invoicing, customer service, etc. The tables in figure 6.9 illustrate how the geography factors influence the relative shares of the different cost groups. We have sorted the companies according to the average value of their geography variables, where the average for each company have been converted to a number between 0 and 1 by dividing them by the value of the HV-variable for that company. We see that companies with a high average value for the geography variables tend to have relatively high operating costs relative to other costs, while the opposite is true for losses.

Upper percentile	No. of companies	Depre-ciation	Interest	Operating costs	Losses	VOLL
100 %	127	20 %	18 %	49 %	10 %	3 %
40 %	51	20 %	17 %	52 %	8 %	4 %
20 %	26	20 %	16 %	52 %	8 %	3 %
10 %	13	18 %	16 %	56 %	7 %	2 %
5 %	6	16 %	16 %	59 %	7 %	2 %

(a) 2005 data set

Upper percentile	No. of companies	Depre-ciation	Interest	Operating costs	Losses	VOLL
100 %	127	19 %	16 %	46 %	15 %	3 %
40 %	51	19 %	15 %	50 %	12 %	4 %
20 %	26	19 %	15 %	50 %	12 %	5 %
10 %	13	18 %	15 %	54 %	10 %	3 %
5 %	6	16 %	15 %	55 %	10 %	4 %

(b) 2006 data set

Figure 6.9: Cost allocation for different levels of Avg(Forest, Snow, Coast) / HV

Another possibility for establishing a limit on the geography factors' share of total cost is to use information from efficiency analyses *without* geography variables. In figure 6.10 below we have sorted the companies according to the value of their geography variables, as in figure 6.9 above, and we show efficiency scores from an analysis which does not include the geography variables. The diagram clearly illustrates the negative correlation between efficiency scores and the geography variables which we observed from the sensitivity analyses in sections 4 and 5. In order to set a limit for the percentage share of the cost norm explained by the geography factors, we could use information concerning a subset of the companies with highest values for the geography variables. The company with the highest value for the geography variables[22] is also the company with the lowest efficiency score, equal to 49.8 %. The inefficiency of 51.1 % experienced by this company could be the result of several causes, including geography-related causes, and could serve as an upper bound on the cost share explained by the geography factors. We also show a cumulative efficiency score for each company, computed as the cost weighted average efficiency for all companies with an average level of the geography factors larger than or equal to the company in question. If we look at the four companies

[22] Austevoll.

with highest value for the geography variables[23], they have a cost weighted efficiency score of 65.5 %, which would translate into a limit for the virtual geography weight of approximately 35 %.

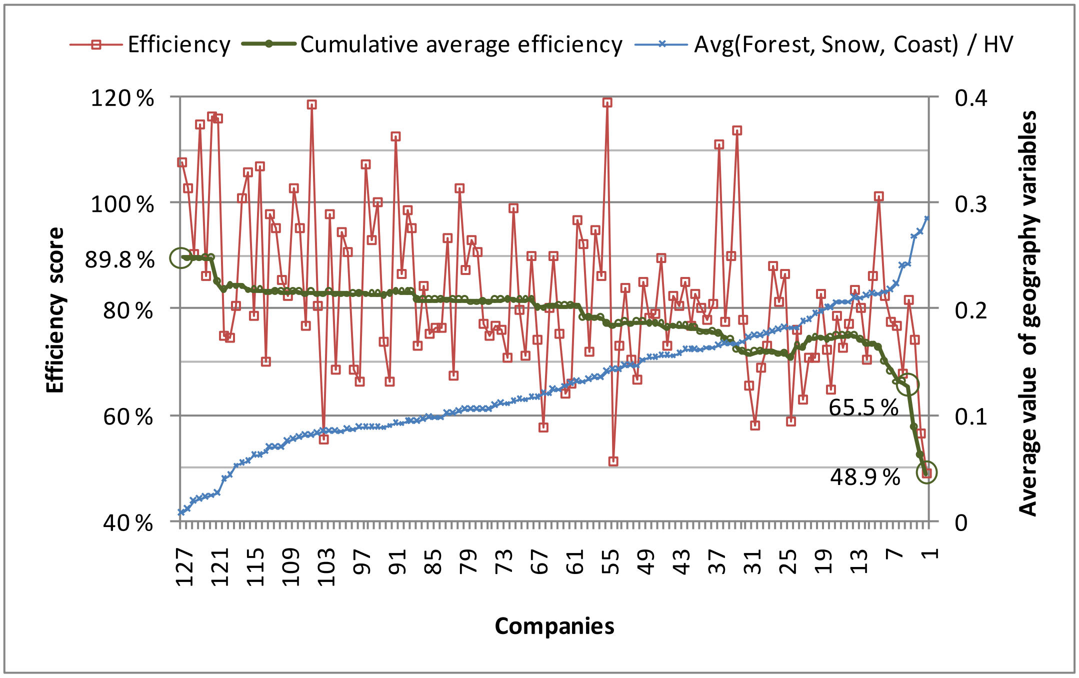

Figure 6.10: Efficiency scores from a model without geography variables (2006)

6.3 Summary

In our opinion it should be possible to establish limits on the total share of the cost norms that can be attributed to geography variables. However, this is a question that should be further pursued, in dialogue with the industry.

[23] Austevoll, Tysnes, Nesset and Trollfjord.

7. Other implementation issues

The DEA field has seen a large growth in recent years, both in terms of theoretical research and applications. This motivated the development of software specifically dedicated to solve DEA models, although most of the general-purpose mathematical programming software can be adapted to solve DEA problems. Examples of program code for a DEA model is described in Olesen and Petersen (1996) for GAMS modeling language and in Emrouznejad (2005) for SAS/OR. These codes can be easily adapted to suit different DEA model formulations. At present, there are several alternative DEA softwares available, including commercial and non-commercial packages. All have good user interfaces and allow interoperability with other applications to read data and export results. State-of-the-art theoretical developments in the DEA theory tend to be implemented relatively fast in advanced modeling options available in a few DEA softwares. Therefore, software modeling capability is usually not a limitation to practical DEA assessments. A recent review of DEA software can be found in Barr (2004), although the features of the software at the time of that review are different from the latest versions available in 2008. Another DEA software review is available in Herrero and Pascoe (2002).

In this section we will provide a summary overview of the capabilities of the commercial and non-commercial DEA software in the versions available in 2008. We will only consider the aspects that are more likely to be needed in this project:

- Type of weight restrictions available
- Estimation of super-efficiency scores
- Estimation of Malmquist indices
- Possibility to select companies to include/exclude in the assessment

The software packages considered in figure 7.1 are those included in Barr (2004), except the ***Pioneer*** (http://faculty.smu.edu/barr/pioneer/) and **EMQ OnFront** (http://www.on-focus.co.kr/econo06.asp), as we could not find any information available in the Internet.

The homepages of the software are:

- DEA Solver Pro (http://www.saitech-inc.com/Products/Prod-DSP.asp)
- Frontier Analyst (http://www.banxia.com/frontier/index.html)

- PIM-DEA Software (http://www.deasoftware.co.uk/). This software corresponds to the new version of the Warwick DEA software, which was renamed, since the developers moved to Aston University, UK.
- DEA Excel Solver (http://www.deafrontier.com/othermodels.html)
- DEAP (http://www.uq.edu.au/economics/cepa/deap.htm)
- EMS (http://www.wiso.uni-dortmund.de/lsfg/or/scheel/ems/)

	Commercial software			Non-commercial software		
	DEA Solver Pro	Frontier Analyst	PIM-DEA Software	DEA Excel Solver	DEAP	EMS
From:	SAITECH	BANXIA	Aston Univ.	J. Zhu	T. Coelli	O. Scheel
Absolute weights	?	?	yes	?	no	no
Assurance regions / Relative weights	yes	no	yes	yes	no	yes
Virtual weights	yes	yes	yes	?	no	no
Superefficiency	yes	yes	yes	yes	no	yes
Malmquist index	yes	yes	yes	yes	yes	yes
Select companies to assess	no	yes	yes	?	no	yes

Figure 7.1: Software overview

Figure 7.1 shows that the PIM-DEA Software, developed by Emmanuel Thanassoulis and Ali Emrouznejad from the Aston University has available all the features that are anticipated to be required for this project.

8. Conclusions

In this report we have studied weight restrictions in the DEA model for distribution networks. The starting point is the NVE model with a single input, total cost, and various outputs, representing delivered energy, the number of customers, and a number of other variables connected to the size and structure of the network and the geographic and climatic challenges of transporting power in the license areas. When applying the DEA model to the data of the distribution companies for 2005 and 2006, we notice large differences in absolute and relative shadow prices, and extreme weight on “geography” variables, especially for small companies. Moreover, it seems to be a tendency that companies, that have a large weight on geographic variables and / or a low weight on transported energy and customers, become super efficient. This seems unreasonable, and one remedy may be to restrict prices / weights for individual outputs, or combinations of outputs, through weight restrictions.

There are various ways to impose weight restrictions, and we consider absolute, relative and virtual weight restrictions with respect to the NVE single input DEA model. We show how to formulate the LP problems and how to interpret the restrictions. We discuss the relative price restrictions suggested for geography and high voltage variables by NVE (2008), and propose a reformulation of the geography variables to make them easier to interpret, and comparable to HV lines. Moreover, we consider an alternative approach, using virtual weight restrictions on the combination of the three geography variables, forest, snow, and coast. Comparing the effects of the virtual approach to the relative, we notice that with relative weight restrictions, more companies are affected, but to a lesser extent. Moreover, we consider combining the maximum on virtual weight restrictions on geography variables with minimum virtual restrictions on delivered energy and customers. Both variants reduce super efficiency in the results. The positive correlation that exists between efficiency scores and the virtual weight on the geography variables in the unrestricted case decreases as a result of the restrictions.

An important task when introducing weight restrictions in the DEA analysis is to determine the specific limits on the weights. Finding reasonable limits, depends on which type of weight restrictions that are in consideration, and should be based on knowledge of cost and technology in the industry. It is an issue that needs to be worked on further, and we recommend doing it in cooperation with the industry. Concerning the choice of type of weight restrictions, we have shown through the analysis of the data, that different methods have different impacts on the efficiency results. An advantage of the virtual weight restrictions on combinations of outputs is that they are on a more aggregated level than the relative ones, implying that the regulator does not have to go into so much detail. This may be an advantage if outputs represent “more than themselves”, for instance by being “proxy” measures on certain cost drivers, or if they represent other correlated factors that are not included in the model. In practice, it may be easier to agree on overall effects on the total cost norm from a subset of outputs, rather than reasonable comparisons of two and two outputs.

Finally, the report discusses implementation of DEA models with weight restrictions, and gives a short overview of available software and their elements.

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